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Liquidity, Government Bonds and Sovereign Debt Crises

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Abstract

This paper analyzes the Eurozone financial crisis through the lens of sovereign bond liquidity. Using novel data, I show that repo haircuts on peripheral government bonds sharply increased during the crisis, reducing their liquidity and amplifying the rise in their yields. I study the impact of this liquidity shock on asset prices and macroeconomic variables in a general equilibrium model with financial frictions calibrated for Ireland. The model confirms the rise in the required returns of illiquid government bonds and predicts a substantial drop in economic activity and deflation. Unconventional policy alleviates the effect of the liquidity shock.

Keywords: repo, haircuts, liquidity shock, funding constraint, unconventional policy

Jel codes: E44, E58, G12

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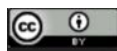
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1 Introduction

“Italian bonds are in the perfect storm at the moment. Real money investors are running away and those using Italian bonds to finance will also be clearing the desk now”.

Financial Times 9th November 2011, “LCH Clearnet SA raises margin on Italian bonds”

Why did countries at the the periphery of the Eurozone (Greece, Ireland, Italy, Portugal and Spain) pay higher interest rates on public debt than countries in the core during the recent financial crisis? Because the creation of a monetary union has integrated the sovereign debt markets and eliminated the exchange rate risk, two main factors may explain this: credit risk and liquidity.

Credit risk derives from the government’s probability of default. The weak fiscal and macroeconomic fundamentals of a country induce investors to ask higher compensation for holding government debt because of the possibility of suffering losses. In addition, fears of default and self-validating expectations may also drive up yields of government securities issued by those countries that cannot print new currency, as predicted by [Calvo \(1988\)](#), [Cole and Kehoe \(2000\)](#) and [Corsetti and Dedola \(2016\)](#).

Liquidity is a broad concept, referred to in the traditional theories of [Keynes \(1936\)](#) and [Hicks \(1962\)](#) as the capacity of an asset to store wealth and protect its owner from a shortage of revenue, thus providing a means to smooth consumption. Modern corporate finance distinguishes between market liquidity and funding liquidity. Market liquidity is the facility to obtain cash by selling an asset; when frictions in the secondary market make it difficult to find a buyer the market liquidity is low and the price of the asset deviates from its fundamentals.¹

¹[Favero, Pagano, and von Thadden \(2010\)](#) and [Manganelli and Wolswijk \(2009\)](#) disentangle

This paper instead focuses on the role of funding liquidity, which is the ease with which investors can obtain funding ([Brunnermeier and Pedersen \(2009\)](#)). As investors typically borrow against an asset, funding liquidity is considered the ability of an asset to serve as collateral. I show that government bonds are the prime collateral securities in the European market of repurchase agreements (repos), which are becoming an essential source of funding for the banking system,² especially since the onset of the crisis when the increase in counterparty credit risk led to a shift from the unsecured to secured funding. This forced borrowing banks to post securities in the interbank market, whose value exceeds the loan by a certain amount, the “haircut” (also called “initial margin” or “margin requirement”), which is the metric that I employ to measure funding liquidity. Given the value of an asset, the lower the haircut the larger the amount of cash that the borrower can obtain by pledging the asset.

Prior to the crisis, the perceived safety of government bonds made them good collateral to back banks’ debt, their repo haircuts were low and their function as a medium of exchange compressed their yields. Nevertheless, I show that during the crisis the emergence of sovereign risk led to rises in repo haircuts on peripheral government bonds, reducing their liquidity and capacity to serve as collateral for secured borrowing. The funding of investors shrank along the lines of the mechanism emphasized by [Gorton and Metrick \(2012\)](#) for the US liquidity crisis in 2007 - 2008, leading to a drop in investment. In order to reduce the contraction of their funding, leveraged investors shifted their portfolios towards the more liquid bonds

the impact of credit risk and market liquidity risk on the evolution of European government bond yields.

²[Gennaioli, Martin, and Rossi \(2014\)](#) find empirical evidence that in a large panel of countries banks hold a sizable amount of government bonds because of their liquidity services.

of the core, with lower haircuts, contributing to the widening of the yield spreads.³

In order to assess this conjecture, I analyze empirically the response of 10-year Irish government bond yield to rises in the haircuts applied by LCH Clearnet Ltd, the largest European clearing house. I identify a funding liquidity shock using the narrative approach, by reading the circulars published by LCH Clearnet Ltd concerning variations in haircuts; I employ the high frequency identification method to address the issue of simultaneity between news in financial markets and the haircut policy of the clearing house. The key identification strategy hinges upon the delay between the announcement and the implementation of changes in haircuts, which implies that haircuts respond to movements in financial variables with one lag, but variations in haircuts affect financial variables instantaneously. I also provide evidence that the funding liquidity shock is not anticipated by market participants.

The impulse responses of vector autoregression and local projection models exhibit a significant increase in government bond yield following a liquidity shock. These results suggest that the returns on a security incorporate a “funding liquidity premium”, in line with [Bartolini, Hilton, Sundaresan, and Tonetti \(2011\)](#), who find that differences in the collateral values across asset classes contributes to explain yield spreads in the US; [Gârleanu and Pedersen \(2011\)](#), who show the emergence of a basis between a security and a derivative with the same cash flow but with different margin requirements; and [Ashcraft, Gârleanu, and Pedersen \(2010\)](#), who

³Banks could alternatively pledge government bonds for ECB refinancing operations, especially after the introduction of fixed-rate full allotment tender procedures, but paying a higher interest rate than the rate in the private repo market (see [Mancini, Ranaldo, and Wrampelmeyer \(2016\)](#)). This opportunity cost of borrowing from the ECB may explain why the share of government bonds on total pledged assets vis-a-vis the ECB diminished during the crisis ([Cassola and Koulischer \(2016\)](#)).

show the inverse relation between haircuts in repos with the Federal Reserve and prices of the underlying collateral during the crisis.

Furthermore, this paper addresses the following question: How does a funding liquidity shock propagate to the real economy? I feed the funding liquidity shock into a DSGE model with financial frictions calibrated for Ireland, building on [Del Negro, Eggertsson, Ferrero, and Kiyotaki \(2017\)](#). Similarly to European banks, investors choose to hold sovereign bonds as a way to store liquidity with which finance future investment. Since they cannot pledge the future returns of investment completely, the liquidity of their asset portfolio is crucial to determine the amount of investment that can be funded. Thus, even if the returns on public bonds are lower than those on private assets, investors can ease their funding constraint by borrowing against them.⁴ Nevertheless, a negative liquidity shock can suddenly reduce the sovereign bond liquidity.

Several papers incorporate the liquidity friction proposed by [Kiyotaki and Moore \(2012\)](#) into general equilibrium models, such as [Kurlat \(2013\)](#), [Bigio \(2015\)](#), [Shi \(2015\)](#), [Ajello \(2016\)](#), [Cui \(2016\)](#), [Del Negro, Eggertsson, Ferrero, and Kiyotaki \(2017\)](#).⁵ These authors assume that privately issued assets are subject to a resaleability constraint, which limits their liquidity, whereas government bonds are perfectly liquid. I depart from this assumption and introduce two types of government bond with different degrees of liquidity, in the spirit of [Hicks \(1939, pag.](#)

⁴This echoes [Hölmstrom and Tirole \(1998\)](#), who show that firms that cannot pledge any of their future income are willing to pay a premium on assets that are able to store liquidity and help them in a state of liquidity shortage. In [Aiyagari and McGrattan \(1998\)](#) government debt enhances the liquidity of households by loosening borrowing constraints. In [Ashcraft, Gârleanu, and Pedersen \(2010\)](#) less liquid assets with higher haircuts are associated with higher returns because investors can borrow less by pledging these assets and need to use more capital.

⁵[Calvo \(2012\)](#), [Gertler and Kiyotaki \(2010\)](#) and [Jermann and Quadrini \(2012\)](#) propose alternative ways to embed liquidity in macroeconomic models.

146): long-term bonds that are subject to a liquidity constraint and short-term bonds that are not. An exogenous liquidity shock is a tightening of the constraint on long-term bonds, which increases the premium that investors are willing to pay for holding short-term bonds. While those papers interpret the liquidity shock as a change in market liquidity and a dry-up of liquidity in the secondary market, in this model it is equivalent to a rise in the repo haircut capturing a change in funding liquidity, akin to [Gârleanu and Pedersen \(2011\)](#) and [Ashcraft, Gârleanu, and Pedersen \(2010\)](#).

The liquidity shock reduces the value of long-term bond and increases its yield to maturity, consistent with the empirical evidence. In addition, it has a quantitative large effect on macroeconomic variables and aggregate output and price level fall, as observed in the data for the Irish economy during the crisis. In particular, investment drops sharply since the liquidity shock lowers the amount of funding that investors can obtain by pledging government bonds. Further, nominal frictions and the zero lower bound (ZLB) prevent the real interest rate from becoming negative and consumption declines.

Finally, I analyze an unconventional policy to alleviate the contractionary effect of the liquidity shock, since conventional monetary policy is constrained by the ZLB. This policy consists of swapping illiquid government bonds for highly liquid papers (short-term debt or money) through direct purchases or collateralized loans in response to the liquidity shock. The liquidity friction in long-term bonds breaks the irrelevance principle of [Wallace \(1981\)](#) and [Eggertsson and Woodford \(2003\)](#) for open market operations, since the government exchanges liquid assets for illiquid assets, thus modifying the composition of aggregate portfolio holdings

and mitigating the drop in investment and output.⁶ The size of the policy intervention is calibrated to match the ECB intervention on the Irish bond market with the expanded asset purchase programme (APP). Results suggest that this policy is more effective in reducing the drop in output with a stronger action.

The structure of the paper is as follows. Section 2 provides a picture of the European repo market and the funding liquidity of peripheral government bonds during the crisis. Section 3 examines empirically the impact of rises in repo haircuts on government bond yield. Section 4 presents the model. Section 5 shows the calibration and the results of the numerical simulations and Section 6 concludes.

2 Funding liquidity of government bonds

This section analyzes the European repo market during the crisis; it presents evidence of the importance of liquidity services provided by sovereign bonds and illustrates how the liquidity of peripheral government securities suddenly dried up.

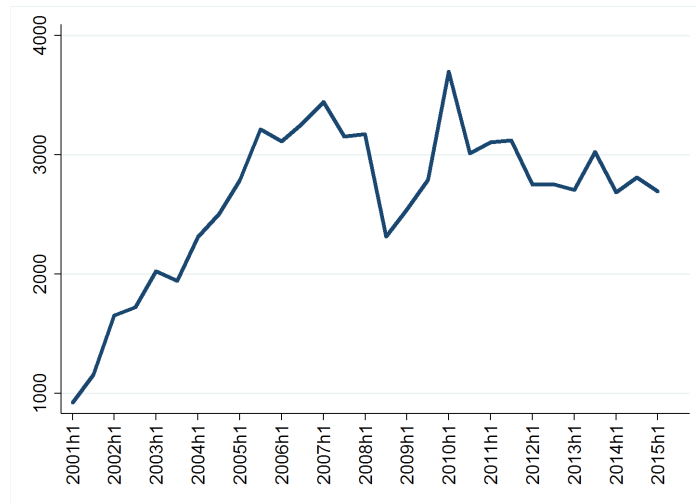
Figure 1 exhibits the extraordinary expansion of the European repo market in the last decade as reported by the the European Repo Market Survey.⁷ Repos tripled in the run-up to the crisis and, after a short contraction between 2008 and 2009, recovered to their pre-crisis level, reaching around €3 trillion. The size of

⁶Although this intervention could be interpreted as the Operation Twist implemented by the Federal Reserve in 1961 and 2011, the objective of this policy is to modify the liquidity of the portfolios of the private sector rather than their maturity. [Chen, Curdia, and Ferrero \(2012\)](#) introduce limits to arbitrage and market segmentation between short-term and long-term bonds in the preferred habitat framework. [Reis \(2017\)](#) also evaluates the effect of Quantitative Easing, assuming that short-term bonds are more liquid than long-term bonds, since they can be used as collateral in the interbank market together with reserves. In his model the unconventional monetary policy relaxes the constraint of banks by exchanging illiquid long-term bonds for liquid reserves.

⁷See Appendix A.1 and A.2 for the definitions employed for repo contracts and a description of the data sources.

the European repo market is therefore considerable and close to that of the US market⁸

Figure 1: European repo market (billions of euros)



Note: The figure shows the amount of repos in the liabilities of banks that participated continuously in all the surveys (billions of euros), excluding repos transacted with the European Central Bank as part of official monetary policy operations.

Source: European Repo Market Surveys (ICMA).

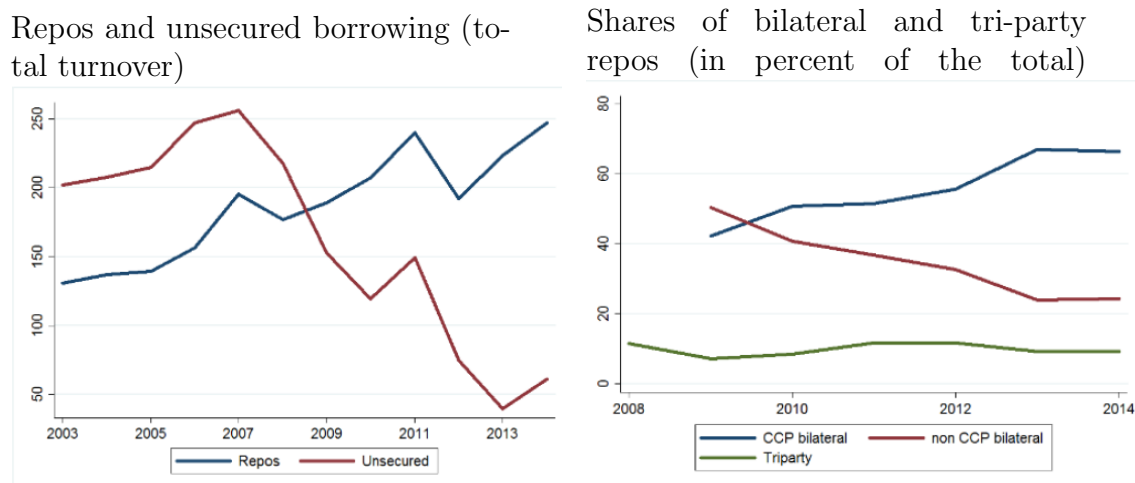
Figure 2 compares the dynamics of secured and unsecured borrowing in interbank transactions using data from the European Money Market Survey of the European Central Bank. There is a massive shift of banks' funding from the unsecured to the secured segment, in particular after the onset of the global financial crisis following the rise in counterparty credit risk.⁹

Furthermore, breaking down the repo market by types of arrangements we

⁸Acharya and Öncü (2012) estimate the size of the US repo market to be about \$4.4 trillion in 2009 based on the average daily amount outstanding of the primary dealers repo financing.

⁹Looking at banks' balance sheets, Table 3 in the Appendix ?? shows that repos are a considerable share of European banks' funding, especially for the largest financial institutions accounting for between 9% and 14% of total liabilities, more than unsecured interbank deposits and long-term debt.

Figure 2: Interbank transactions in the European money market



Note: The figure shows the evolution of repos and unsecured borrowing in the European Money Market as total turnover (left panel), and the shares of bilater CCP-cleared repos, bilateral over-the-counter repos and tri-party repos in percent of total repos (right panel).

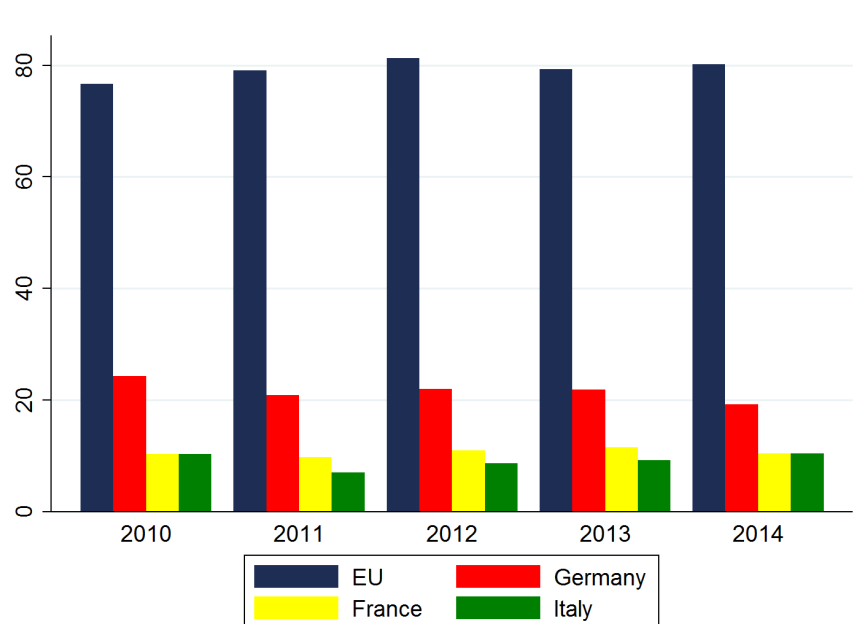
Source: European Money Market Surveys (ECB)

can observe that bilateral CCP-cleared repos steadily increased, while over-the-counter bilateral repos declined. Tri-party repos account for a little share of the market, in contrast to the US repo market where they are the largest component. The enhanced role of clearing houses increases the importance of the quality of collateral securities since they set repo haircuts as a function of the credit risk of these securities. Thus, a change in the credit risk is reflected in variations in haircuts affecting funding conditions in the European repo market.

Concerning the collateral composition, Figure 3 shows that government bonds are the predominant securities, accounting for around 80% of the total collateral pool. This share was stable during the crisis and represents a structural characteristic of the European repo market, different to the US market where securities

issued by the private sector account for a larger share (Krishnamurthy, Nagel, and Orlov (2014)). Looking at the composition of sovereign securities, German bonds are the largest share, although their supply is lower than French and Italian bonds (Eurostat (2013)). In addition, the share of Italian bonds dropped substantially during the Eurozone crisis, from 10% in December 2010 to 7% in December 2011.

Figure 3: Share of collateral in the European repo market (in percent of the total)



Note: The figure shows the share of collateral securities in the European repo market (in percent of the total).

Source: European Repo Market Surveys (ICMA).

The collateral composition of the European repo market reflects not only the safety of securities but also their liquidity. Figure 4 shows the evolution of 10-year government bond yields of peripheral countries (Ireland, Portugal, Italy and Spain) and the haircuts applied on these securities by LCH Clearnet, the largest European clearing house. Following the rise in bond yields and sovereign risk, haircuts on

Irish and Portuguese sovereign securities surged up to 80%, making those bonds almost completely illiquid. Italian and Spanish bonds also experienced rises in haircuts but these were more mitigated. However, it is also interesting to note that the increase in haircuts on Italian bonds is associated with their decrease in the share of collateral in the repo market (Figure 3).¹⁰

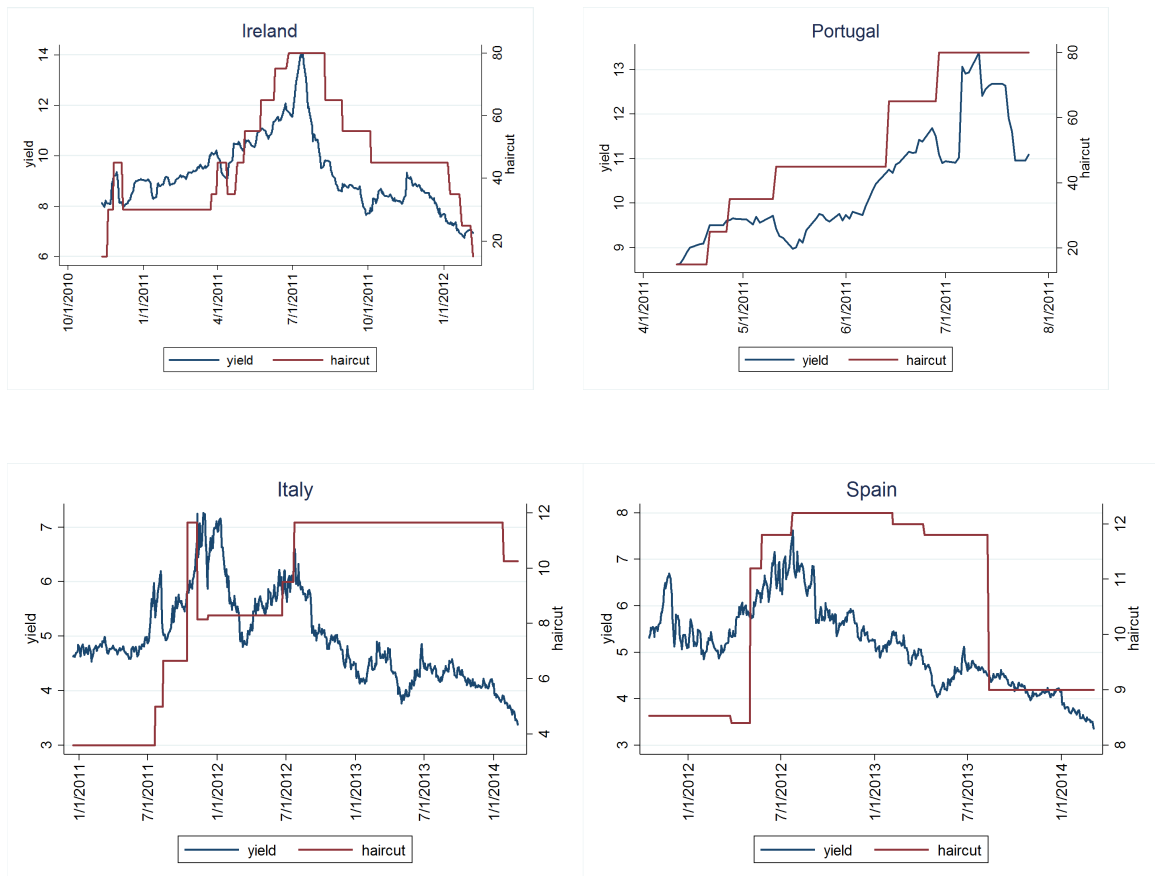
To summarize, European government bonds have become an essential liquid instrument for banks, especially since the onset of the financial crisis, because they are needed to pledge collateral securities as guarantee of repayment in order to borrow on the interbank market. Therefore, their value incorporates a premium reflecting their capacity to serve as collateral. The increase in haircuts on peripheral bonds reduced their liquidity premium, leading to a flight-to-liquidity from the periphery to the core and increasing their required returns, which in turn entailed additional rises in haircuts by clearing houses.

In the next section, I investigate empirically this liquidity spiral in the Irish economy; the rich dynamics of the haircut applied by LCH Clearnet makes it a good laboratory to study its impact on government bond yield. Furthermore, [Delatte, Fouquau, and Portes \(2017\)](#) report the sell-off in Irish bonds during the crisis driven by higher collateral requirements, quoting the following article from *Financial Times* (“Irish Bond Yields Leap after Selling Wave”, November 10, 2010): “The dramatic sell-off in Irish bonds was driven by a fire sale of positions by market participants who were unable to meet collateral requirements enforced by LCH Clearnet - one of the biggest clearing house - on Wednesday morning. Ireland’s bank were faced with an estimated \$1 bn cash-call from LCH Clearnet

¹⁰In [Arnakola, Douady, Laurent, and Molteni \(2016\)](#) we show that a similar pattern is observed for haircuts applied by other major European clearing houses and the volume of repos is concentrated in a few large clearing houses.

as a result of its decision to require a deposit of 15 percent against all Irish bond positions as indemnity against default. [...] In order to avoid the call, many other banks and traders are dumping their bond positions, however.”

Figure 4: Yields (LHS) and haircuts (RHS) on 10-year government bonds



Note: The figure shows the evolution of 10-year government bond yields and haircuts applied on these securities by LCH Clearnet for Ireland (top-left), Portugal (top-right), Italy (bottom-left) and Spain (bottom-right).

Source: Bloomberg and LCH Clearnet website.

3 Assessing the impact of a funding liquidity shock

I study the dynamic relationship between the haircut and Irish government bond yield with a high-frequency Bayesian vector-autoregression (BVAR) model, including the series of haircuts applied by LCH Clearnet Ltd on 10 year government bonds (h_t), the logarithm of 5-year sovereign CDS spread (CDS_t) and the logarithm of yield of 10-year government bonds (yd_t), in order to deal with endogeneity problems and reverse causality issues because CDS spread and government bond yield spread over the German bonds are part of the information set used by this clearing house to settle the level of haircuts.¹¹ The inclusion of the CDS spread as a measure of credit risk allows to disentangle the funding liquidity channel from the credit risk channel and helps identify a funding liquidity shock. Data on CDS spread and government bond yield come from Datastream and Bloomberg, respectively.

The sample covers the period from 11/01/2010 to 12/01/2011 at daily frequency. The sample size is due to the data availability on changes in haircuts and is right censored to exclude the launch of the first 3 year LTRO on 8th December 2011. Other unconventional monetary policies, such as the Security Market Programme (SMP) and the refinancing operations with full allotment, were already active during this period, but their inception, which is likely to have an impact on market sentiments, precedes the beginning of the sample.¹²

This exercise is close to the study of Pelizzon, Subrahmanyam, Tomio, and Uno

¹¹See LCH. Clearnet (2011): http://www.ecb.europa.eu/paym/groups/pdf/mmcg/Item_1_LCH_Margining.pdf?0fe79f1cef93461dc22566a4e165db44.

¹²Ghysels, Idier, Manganelli, and Vergote (2017) find that the SMP was effective in reducing yields of peripheral government bonds. Because of the scarcity of public available information regarding the implementation of the SMP, I cannot control for it but we can expect that this policy has dampened the impact of variations in haircuts on government bond yields.

(2016), who analyze the dynamic relation between credit risk and market liquidity on the Italian sovereign debt market. The authors find that this link is reinforced above 500 basis points in the sovereign CDS spread, because of changes in haircuts applied by LCH Clearnet. They also show that the threshold effect disappears after December 2011, when the ECB started the 3-year LTROs, providing abundant liquidity to the banking system, and conducted moral suasion to the European clearing houses to limit the procyclicality of haircuts, loosening the link between credit risk and market liquidity.

Let $y_t = [h_t, CDS_t, yd_t]'$ the vector of endogenous variables and consider the reduced form VAR

$$y_t = A_c + A_1 y_{t-1} + \dots + A_p y_{t-p} + u_t \quad (1)$$

where A_j are 3 x 3 matrices of coefficients with p denoting the number of lags, and u_t is the 3 x 1 vector of reduced form residuals with $u_t \sim (0, \Sigma_u)$. I set $p = 4$ but results are robust to a longer lag length. I adopt non-informative priors for estimating the matrices of coefficients and covariance.¹³ Since the components of u_t may be instantaneously correlated (i.e. Σ_u could be not diagonal), I consider the following structural VAR model with orthogonalized residuals

$$A_0 y_t = A_c^* + A_1^* y_{t-1} + \dots + A_p^* y_{t-p} + \epsilon_t \quad (2)$$

where A_0 is the impact matrix, $A_j^* = A_0 A_j$, and ϵ_t are the structural shocks with diagonal covariance matrix Σ_ϵ . Reduced form residuals can be expressed as a linear combination of structural shocks $u_t = A_0 \epsilon_t$ and $\Sigma_u = A_0 \Sigma_\epsilon A_0'$. Normalizing the

¹³Appendix B.2 reports the details of the estimation procedure.

variances of the structural shocks to one (i.e. $\mathbb{E}(\epsilon_t \epsilon_t') = I$) gives

$$\Sigma_u = A_0 A_0'$$

In order to impose the necessary restrictions on A_0 to achieve the identification of structural shocks, I apply a Cholesky decomposition of Σ_u , choosing \tilde{A}_0 as a lower triangular matrix with positive elements on the main diagonal. The recursive structure and zero restrictions on the contemporaneous coefficients find justification in the procedure through which LCH Clearnet Ltd decides and communicates to its members variations in haircuts, which is key to identify a funding liquidity shock. LCH Clearnet Ltd notifies all modifications in haircuts at the close of business through the Repo Clear Margin Rate Circulars; so their revision is applied one day after the publication of the Circulars. To give a sense of how I proceed, Figure 13 in the Appendix B.1 reports an example of the Circular; these documents provide information on the date of the announcement, date of implementation and variations in additional margins required.

Thus, I place the haircut as the first variable in the BVAR model. This Choleski ordering implies that the haircut does not respond within the period of impact to financial shocks relative to CDS spread and yield, but a shock to the haircut is allowed to affect CDS spread and yield instantaneously. This identification strategy is based on a similar assumption employed in the empirical literature on fiscal policy; that is, fiscal instruments do not react instantaneously to variations in macroeconomic variables, mainly economic activity, because of the outside lag, which is the delay between the decision and the implementation of a certain pol-

icy.¹⁴

Nevertheless, with low frequency data the implementations of fiscal policy can be anticipated by private agents, leading to a non-fundamental moving average representation (see [Leeper, Walker, and Yang \(2013\)](#), [Ramey \(2011\)](#), [Mertens and Ravn \(2012\)](#), [Favero and Giavazzi \(2012\)](#)). In this application high frequency data rule out the possibility that market participants may react to announcements of changes in haircuts before their implementation. In this regard, it shares the High Frequency Identification (HFI) approach for monetary policy shocks.¹⁵

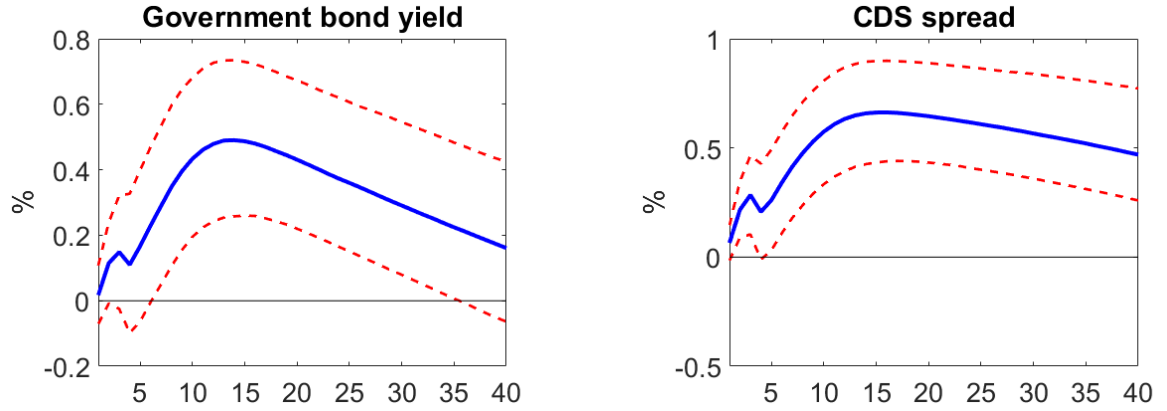
Figure 5 presents the impulse responses to one standard deviation shock to haircut. The solid line denotes the median estimate of the impulse responses and the dashed lines represent the range of the 90-percent confidence band around the point estimates. The funding liquidity shock leads to a rise in government yield which remains significant for around 35 days with the peak effect to 0.5 percent after 14 days from the impact period. The funding liquidity shock also raises the CDS spread, with the peak effect to 0.66 occurring after 16 days; the impact on the CDS spread is more persistent than on yield. These results suggest that the higher haircuts set by LCH Clearnet on Irish government bonds increased the yield of government bonds and their credit risk as measured by CDS spread, which in turn led to additional increases in haircut by the clearing house.

A possible issue for the identification of a funding liquidity shock is that market participants can anticipate the decision of the clearing house to change the haircuts.

¹⁴Using different approaches [Blanchard and Perotti \(2002\)](#), [Fatás and Mihov \(2001\)](#) assume that government spending is predetermined within the quarter imposing time restrictions in a SVAR. [Romer and Romer \(2010\)](#) and [Ramey \(2011\)](#) also place their narrative fiscal measures as first endogenous variable with a Choleski decomposition.

¹⁵The HFI approach addresses the problem of simultaneity which arises with low frequency data since within a period monetary policy both affects financial variables and responds to their movements (See [Ramey \(2016\)](#) for a survey).

Figure 5: Impulse response function of a liquidity shock



Note: The figure show the responses of 10-year government bond yield (left panel) and CDS spread (right panel) to one standard deviation shock to haircuts. The solid blue line plots the posterior median and the red dash lines are the 10th and 90th percentiles with 50000 draws for which the first 20000 are discarded as burn-in draws.

In particular, LCH Clearnet Ltd published indicative thresholds at a 450 basis points spread at the 10-year maturity to AAA benchmark, or at a 500 basis points at 5-year CDS spread as risks indicators.¹⁶ However, the clearing house also states that these are key indicators to judge the credit risk of a security but do not trigger automatic actions for increases in haircuts and margin calls.¹⁷ Furthermore, [Bank of Italy \(2013\)](#) documents that changes in haircuts applied by LCH Clearnet partly followed discretionary criteria and were unexpected.

In order to confirm that changes in haircut could not be anticipated, I perform two statistical tests. First, I construct a variable h_t^a for the changes in haircuts

¹⁶See [LCH. Clearnet \(2010\)](#): http://secure-area.lchclearnet.com/member_notices/circulars/2010-10-05.asp and [LCH.Clearnet Margining Approach \(2011\)](#): http://www.ecb.europa.eu/paym/groups/pdf/mmcg/Item_1_LCH_Margining.pdf?0fe79f1cef93461dc22566a4e165db44.

¹⁷See [LCH Clearnet \(2014\)](#) “Frequently asked questions on the sovereign risk framework”. Furthermore, [ICMA \(2015, pag. 25\)](#) argues that “Although CCPs apply more rigorous risk management practices than many market users, their methodologies are often proprietary and therefore opaque, and it is not possible for members to scrutinize these methodologies, despite their critical dependence on them”.

on Irish bonds on the date of the announcement of their revision, instead of on the date of the effective changes as measured by h_t . I run a Granger causality test of CDS spread and yield spread to 10-year German bonds on the announced variations in haircut h_t^a . If they help predict variations in h_t^a , market participants could anticipate their modifications by looking at these indicators of sovereign risk. Table 4 in the Appendix B.1 shows that the CDS spread and yield spread fail to predict announced changes in haircuts, confirming that changes in haircuts did not automatically follow variations in these indicators and were in part discretionary. Second, I run the Hansen (2000) test to assess the presence of a threshold regressing h_t on CDS_t .¹⁸ Figure 14 in the Appendix B.1 displays the graph on the normalized likelihood ratio sequence as a function of the threshold in CDS spread. The graph provides evidence of a threshold at 562 basis points, substantially higher than the 500 basis points threshold published by LCH Clearnet Ltd, as a key indicator of risk. These tests suggest that variations in haircuts were to a large extent exogenous liquidity surprises.

As a robustness test, I estimate impulse response by local projections (Jordà (2005)), using the series of haircuts as narrative shock. This approach does not require transforming the VAR model into a vector moving average model for the impulse response function, using estimated parameters for horizon 0 to iterate forward. Moving average representation of VAR can be non-fundamental with small-scale VAR or in the presence of anticipated shocks, so the advantage of using an observable narrative shock is that it allows to compute impulse responses omitting a large amount of information that would be orthogonal to the shock

¹⁸I obtain similar results when I regress h_t^a on CDS_t . The yield spread to 10-year German bonds was above 450 basis points for the whole period of the sample.

included in the regression. The model is the following

$$yd_t = \boldsymbol{\delta}' \mathbf{X}_{t-1} + \beta shock_t + u_t \quad (3)$$

where \mathbf{X}_{t-1} include a constant and the first four lags of the variables CDS_t and yd_t ; $\boldsymbol{\delta}$ collects the coefficients; $shock_t$ corresponds to the series of haircut.¹⁹ I use the Newey-West corrections for standard errors because of the serial correlation in the error terms induced by the successive leading of the dependent variable. Figure 6 shows that with local projections the funding liquidity shock has a substantial effect on government bond yield. A one percent shock in the haircut increases the government bond yield with the peak effect to 4.8 percent after 18 days. The impact remains significant for 30 days, analogously to the impulse response estimated with the BVAR.

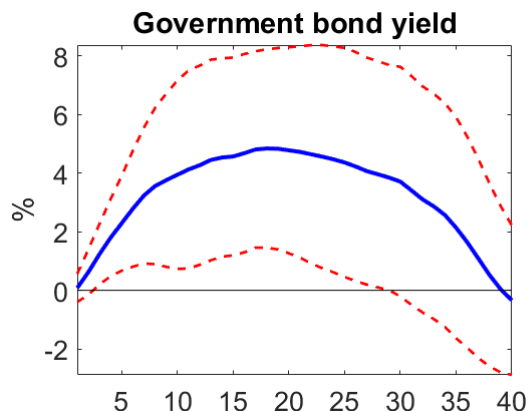
Overall, these findings lend support to the presence of a negative liquidity spiral on the Irish sovereign debt market during the crisis, which amplified the rise in the yields. Since the high frequency of financial series is essential for the identification of a funding liquidity shock, I do not consider its impact on macroeconomic variables in the empirical analysis.²⁰ However, empirical literature provides evidence that, during the crisis, the rise in government bond yields negatively affected firms' capital expenditure through the bank lending channel (see Acharya, Eisert, Eufinger, and Hirsch (2014) and Altavilla, Pagano, and Simonelli (2016)). In the next section I feed the funding liquidity shock identified with the narrative approach into a DSGE model featuring liquidity frictions, in order to study its impact on

¹⁹When I include a linear and quadratic time trend results remain almost unchanged.

²⁰The mixed frequency VAR approach creates a bridge between high frequency financial variables and low frequency macroeconomic variables but cannot keep the daily frequency of financial variables that I exploit for the identification strategy (see Foroni and Marcellino (2014)).

macroeconomic variables.

Figure 6: Response of government bond yield to a liquidity shock



Note: The figure shows the impulse response of 10-year government bond yield to a one percent liquidity shock. The dash lines represent 95 percent bands that are based on Newey-West standard error.

4 The model

The model is an infinite horizon economy populated by a continuum of households of measure one. The members of each household are either entrepreneurs or workers. The model incorporates nominal rigidities, since prices and wages are set in staggered contracts, and real rigidities with capital adjustment cost along the lines of [Christiano, Eichenbaum, and Evans \(2005\)](#) and [Smets and Wouters \(2007\)](#).²¹ Households allocate saving across three riskless financial assets characterized by different degrees of liquidity: equity, long-term and short-term sovereign bonds. Long-term bonds are subject to a liquidity shock which is the only shock perturb-

²¹Appendix C.1 through C.4 reports the equations for the production process, market-clearing and equilibrium conditions.

ing the economy. In response to this shock the government can implement an unconventional policy that consists of increasing the supply of short-term bonds that are more liquid than long-term bonds.

4.1 Households

Structure. Each household has a unit measure of members indexed $j \in [0, 1]$. At the beginning of each period all members are identical and hold an equal share of the household's assets. They receive an idiosyncratic shock, iid across members and across time, which determines their profession. With probability γ they are entrepreneurs and with probability $1 - \gamma$ they are workers. By the law of large number γ also represents the fraction of entrepreneurs in the economy. Each entrepreneur $j \in [0, \gamma)$ invests and each worker $j \in [\gamma, 1]$ supplies labor; both types return their earnings to the household and at the end of each period all members share consumption goods and assets, but resources cannot be reallocated among members within the period.

Preferences. The household's objective is to maximize the utility function

$$\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{c_s^{1-\sigma}}{1-\sigma} - \frac{\xi}{1+\eta} \int_{\gamma}^1 h_s(j)^{1+\eta} dj \right] \quad (4)$$

where \mathbb{E}_t denotes the conditional expectation, β is the subjective discount factor, σ measures the degree of relative risk aversion, ξ is a scaling parameter that can be chosen to match a target value for the steady state level of hours and η is the inverse of the Frisch elasticity of the labor supply. Utility depends positively upon the sum of the consumption good bought by household members ($c_t = \int_0^1 c_t(j) dj$) and negatively upon the workers' labor supply h_t .

Portfolio. Households hold physical capital k_t with price q_t that depreciates for a fraction δ every period. They can sell claims on their capital to other households, s_t^I , which represents the only liabilities of households, and purchase claims on other households' capital, s_t^O . Equity issued by the other households (s_t^O) and the unmortgaged capital stock ($k_t - s_t^I$) are assumed to yield the same returns, have the same value and liquidity and depreciate at the same rate, so they are perfect substitutes and can be summed together and defined as net equity

$$s_t = s_t^O + k_t - s_t^I \quad (5)$$

Households also own government debt with different maturities. Short-term bonds b_t^S are one period securities purchased at time t at price q_t^S that pay a unit return at time $t + 1$. The price of short-term bond is the inverse of the gross nominal interest rate

$$q_t^S = \frac{1}{r_t} \quad (6)$$

Long-term bonds b_t^L are perpetuities with coupons which decay exponentially as in [Woodford \(2001\)](#) with price q_t^L . A bond issued at date t pays λ^{k-1} at date $t + k$, where $\lambda \in [0, 1]$ is the coupon decay factor that parametrizes the maturity of long-term bonds, corresponding to $(1 - \lambda\beta)^{-1}$. If $\lambda = 0$ these securities are one period zero coupon bonds and if $\lambda = 1$ they are consols.²² In the steady state the returns on long-term and short-term bonds are linked by a non-arbitrage condition. Following [Chen, Curdia, and Ferrero \(2012\)](#), the gross yield to maturity at time t

²²An alternative interpretation of the long-term debt is that λ is the fraction of the outstanding bonds paying a constant coupon of 1 and $(1 - \lambda)$ is the fraction of bonds which mature at each period and for which the government pays back the principal to the bond holder ([Chatterjee and Eyigungor \(2012\)](#)).

on long-term bond is defined as

$$yd_t = \frac{1}{q_t^L} + \lambda \quad (7)$$

At the end of each period households receive the dividend per unit of capital ownership r_t^K , which is the sum of rental income of capital and profits of intermediate good producers and capital good producers. Table 1 summarizes the household's balance sheet at the beginning of the period.

Table 1: Household's balance sheet (financial assets)

Asset		Liability	
Capital stock:	$q_t k_t$	Equity issued:	$q_t s_t^I$
Other's equity:	$q_t n_t^O$		
Long-term bonds:	$q_t^L b_t^L / p_t$		
Short-term bonds:	$q_t^S b_t^S / p_t$	Net worth:	$q_t s_t + q_t^L b_t^L / p_t + q_t^S b_t^S / p_t$

Resources allocation. When the asset market and goods market open households allocate their resources and trade assets to finance new investments. The flow of fund constraint of household member j is as follows

$$\begin{aligned}
& c_t(j) + q_t^I i_t(j) + q_t^L \frac{b_{t+1}^L(j)}{p_t} + q_t^S \frac{b_{t+1}^S(j)}{p_t} + q_t [s_{t+1}(j) - i_t(j)] \\
& = [1 + \lambda q_t^S] \frac{b_t^L}{p_t} + \frac{b_t^S}{p_t} + [r_t^K + (1 - \delta)q_t] s_t + \frac{w_t(j)}{p_t} h_t(j) - t_t
\end{aligned} \quad (8)$$

where p_t denotes the price level, q_t^I is the cost of one unit of new capital in terms of consumption goods, differing from 1 because of capital adjustment cost, $w_t(j)$ is the nominal wage for workers j . According to the left side of the budget constraint, the household members allocate resources between purchase of non-storable con-

sumption good, investment in new capital - if they are entrepreneurs - net purchase of equity, long-term bonds and short-term bonds. They finance their activities on the right side of the budget constraint with returns on financial assets (equity, long-term bonds and short-term bonds) and wages - if they are workers - net of taxes.

A key assumption of the model is the presence of the following funding constraints that limit the financing of new investments by entrepreneurs and determine the different degree of liquidity of the assets

$$s_{t+1}(j) \geq (1 - \theta)i_t(j) + (1 - \delta)s_t \quad (9)$$

$$b_{t+1}^L(j) \geq (1 - \phi_t)b_t^L \quad (10)$$

$$b_{t+1}^S(j) \geq 0 \quad (11)$$

Inequality 9 means that the entrepreneur can issue claims on the future output of investment but only for a fraction $\theta \in [0, 1]$. This borrowing constraint implies that investment is partially funded internally and entrepreneurs have to retain $1 - \theta$ as their own equity. In addition, equity is assumed to be completely illiquid since entrepreneurs cannot sell it to obtain more resources to invest. Hence, the entrepreneurs' equity holding at the start of the period $t+1$ must be at least the sum of $(1 - \theta)i_t$ and depreciated equity $(1 - \delta)s_t$, where λ is the inverse depreciation rate.²³

²³Nezafat and Slavik (2017) model a financial shock as a tightening in the credit conditions and a drop in θ and assume that equity/capital is completely liquid. In our set-up the assumption that equity is illiquid means that entrepreneurs cannot issue equity on the unmortgaged capital

The entrepreneur can acquire additional resources by disposing of a fraction $\phi_t \in [0, 1]$ of long-term bonds, so a resaleability constraint is imposed to keep the residual $(1 - \phi_t)$ of bonds in his portfolio (inequality 10). $(1 - \phi_t)$ is equivalent to the haircut in a repo transaction since it determines the amount of liquidity that the entrepreneur can obtain by pledging sovereign securities in secured borrowing. In other words, the entrepreneur cannot borrow against the entire bond holding because of the presence of the haircut.

Inequality 11 implies that short-term bonds are not subject to resaleability constraint and are fully liquid, but entrepreneurs cannot borrow from the government.²⁴ ϕ_t is the key parameter of the model characterizing the liquidity of financial assets. We can think that it takes value 0 for equity, value 1 for short-term bonds and an intermediate value for long-term bonds. The assumption on the diverse resaleability of equity and bonds reflects the different liquidity of privately issued securities, which are scarcely used as collateral, and sovereign bonds, which are largely pledged by European banks for repo transactions, and gives government bonds a special liquidity function. ϕ_t follows a stationary AR(1) process and the dynamic of the model follows an unexpected negative shock to ϕ_t , which is paramount to a rise in the repo haircut on sovereign bonds.²⁵

At the end of the period, the assets of households are given by

$$s_{t+1} = \int s_{t+1}(j) dj \quad (12)$$

stock and cannot sell any of the remaining equity to others.

²⁴Similarly, inequalities 9 and 10 ensure that receipts from trading equity and long-term bonds are strictly positive, which prevents the entrepreneur from going short on these securities.

²⁵The model is solved with the Newton-Raphson algorithm in order to take into account the ZLB and non-linear perfect foresight.

$$b_{t+1}^S = \int b_{t+1}^S(j) \, dj \quad (13)$$

$$b_{t+1}^L = \int b_{t+1}^L(j) \, dj \quad (14)$$

$$k_{t+1} = (1 - \delta)k_t + \int i_t(j) \, dj \quad (15)$$

Next, the specific functions of entrepreneurs and workers are taken into account.

4.1.1 Entrepreneurs

The entrepreneur $j \in [0, \gamma)$ does not supply labor, so $H_t(j) = 0$ in equation 8 to get his budget constraint. In order to acquire new capital he can either produce it at price q_t^I or buy it in the market at price q_t . For the rest of the model I assume that $q_t > q_t^I$ in order to focus on the economy where the liquidity constraints bind, thus limiting the ability of the entrepreneur to finance investments. In this case, entrepreneurs will use all the available liquidity for new investment projects to maximize the households' utility. Accordingly, they minimize the equity holding by issuing the maximum amount of claims on the investment return. The entrepreneur also sells the maximum amount of bonds as allowed by constraints 10 and 11, because their expected returns are lower than those on new investment. As a result, in equilibrium the liquidity constraints all bind and the entrepreneur does not consume goods within the period:

$$s_{t+1}(j) = (1 - \theta)i_t(j) + (1 - \delta)s_t \quad (16)$$

$$b_{t+1}^L(j) = (1 - \phi)b_t^L \quad (17)$$

$$b_{t+1}^S(j) = 0 \quad (18)$$

$$c_t(j) = 0 \quad (19)$$

Given the solutions for entrepreneurs, $s_{t+1}(j)$, $b_{t+1}^L(j)$, $b_{t+1}^S(j)$, c_t for $j \in [0, \gamma)$, equations 16, 17, 18 and 19 can be plugged into equation 8 to derive the function of investment for entrepreneurs

$$i_t(j) = \frac{r_t^K s_t + [1 + \lambda \phi_t q_t^L] \frac{b_t^L}{p_t} + q_t^S \frac{b_t^S}{p_t} - t_t}{q_t^I - \theta q_t} \quad (20)$$

Aggregating by entrepreneurs total investment is

$$i_t = \int_0^\gamma i_t(j) \, dj = \gamma \frac{r_t^K s_t + [1 + \lambda \phi_t q_t^L] \frac{b_t^L}{p_t} + q_t^S \frac{b_t^S}{p_t} - t_t}{q_t^I - \theta q_t} \quad (21)$$

Investment is a function of the the maximum liquidity available for the entrepreneurs (the numerator) and the downpayment, which is the difference between the price of one unit of investment goods q_t^I and the value of equity issued by the entrepreneur θq_t (the denominator).

4.1.2 Workers

Workers $j \in [\gamma, 1]$ do not invest, so $i_t(j) = 0$. They supply labor as demanded by firms at a fixed wage; the union who representing each type of worker sets wages

on a staggered basis. To determine the asset and consumption choices of workers, I first derive the household's decision for $s_{t+1}, b_{t+1}^L, b_{t+1}^S$ and c_t , taking w_t and h_t as given. Knowing the solution for entrepreneurs, $s_{t+1}(j), b_{t+1}^L(j), b_{t+1}^S(j)$ and $c_t(j)$ can be determined for workers, given the aggregate consumption and asset holding.

4.1.3 The problem of households

To solve the model for the household, I aggregate the workers' and entrepreneurs' budget constraint

$$\begin{aligned} & c_t(j) + q_t^I i_t(j) + q_t^L \frac{b_{t+1}^L(j)}{p_t} + q_t^S \frac{b_{t+1}^S(j)}{p_t} + q_t[s_{t+1}(j) - i_t(j)] \\ &= [1 + \lambda q_t^S] \frac{b_t^L}{p_t} + \frac{b_t^S}{p_t} + [r_t^K + (1 - \delta)q_t]s_t + \int_{\gamma}^1 \frac{w_t(j)}{p_t} h_t(j) - t_t \end{aligned} \quad (22)$$

Households maximize the utility function (5) by choosing c_t, s_{t+1}, b_{t+1}^L and b_{t+1}^S subject to the aggregate budget constraint and the investment constraint. The first order conditions for equity, long-term bonds and short-term bonds are respectively

$$U'_{c,t} = \beta \mathbb{E}_t \left\{ U'_{c,t+1} \left[\frac{r_{t+1}^K + (1 - \delta)q_{t+1}}{q_t} + \frac{\gamma(q_{t+1} - q_{t+1}^I)}{q_{t+1}^I - \theta q_{t+1}} \frac{r_{t+1}^K}{q_t} \right] \right\} \quad (23)$$

$$U'_{c,t} = \beta \mathbb{E}_t \left\{ \frac{1}{\pi_{t+1}} U'_{c,t+1} \left[\frac{1 + \lambda q_{t+1}^L}{q_t^L} + \frac{\gamma(q_{t+1} - q_{t+1}^I)}{q_{t+1}^I - \theta q_{t+1}} \frac{1 + \lambda \phi_{t+1} q_{t+1}^L}{q_t^L} \right] \right\} \quad (24)$$

$$U'_{c,t} = \beta \mathbb{E}_t \left\{ \frac{1}{\pi_{t+1}} U'_{c,t+1} \left[r_t + \frac{\gamma(q_{t+1} - q_{t+1}^I)}{q_{t+1}^I - \theta q_{t+1}} r_t \right] \right\} \quad (25)$$

where π_t is the inflation rate defined as $\pi_t = \frac{p_{t+1}}{p_t}$. The choice of sacrificing one unit of consumption today to purchase a paper gives a payoff which is composed of two parts. The first is the returns on the assets: $\frac{r_{t+1}^K + \lambda q_{t+1}}{q_t}$ for equity, $\frac{1 + \lambda q_{t+1}^L}{q_t^L}$ for long-term bonds and r_t for short-term bonds. The second part is a premium, deriving from the fact that papers in the entrepreneurs' portfolio relax their investment constraint. This premium is a function of the leverage $\frac{1}{q_t^L - \theta q_t}$, the gap between q_t and q_t^L , which represents the marginal value of relaxing the constraint, and the liquid returns of each asset. The bond holding eases the financing constraints more than the equity holding, which makes bonds more valuable for entrepreneurs. Further, long-term bonds pay a liquidity premium relative to short-term bonds, which depends on ϕ_{t+1} . The spread between long-term and short-term bond is the difference between the returns of the two assets:

$$spr_t = \frac{1 + \lambda q_{t+1}^L}{q_t^L} - r_t \quad (26)$$

4.2 The Government

The government conducts conventional fiscal and monetary policy, and unconventional monetary policy. The intertemporal budget constraint is

$$q_t^L \frac{b_{t+1}^L}{p_t} + q_t^S \frac{b_{t+1}^S}{p_t} + t_t = (1 + \lambda q_t^L) \frac{b_t^L}{p_t} + \frac{b_t^S}{p_t} \quad (27)$$

The debt repayment is financed by the issue of new debt and net taxes t_t , which can be interpreted as primary balance. A solvency condition links taxes with the outstanding beginning-of-period long-term debt in deviations from the steady state to satisfy the government's intertemporal budget constraint. The government

adjusts fiscal policy when the debt level goes up as in [Chen, Curdia, and Ferrero \(2012\)](#) and [Davig and Leeper \(2006\)](#)

$$t_t - t = \psi_T \left(\frac{b_t^L}{p_t} - \frac{b^L}{p} \right) \quad (28)$$

where $\psi_T > 0$ measures the elasticity of fiscal policy to variations in the size of the debt. Short-term debt does not enter into the solvency condition to avoid that the unconventional policy is counteracted by higher taxes.²⁶ However, quantitatively results do not change by including the short-term debt since the adjustment of taxes to debt is gradual (ψ_T is small) and at the steady state short-term bond is in zero net supply. The government sets the nominal interest rate following the feedback rule constrained by the zero lower bound condition

$$r_t = \max \left\{ r \pi_t^{\psi_\pi} \left(\frac{y_t}{y} \right)^{\psi_y}, 1 \right\} \quad (29)$$

where $\psi_\pi > 1$ and $\psi_y > 0$. Unconventional policy consists of purchasing illiquid long-term bonds by issuing liquid short-term bonds when the liquidity of long-term bonds dries up. The supply of one-period bond is proportional to the deviation of liquidity from its the steady state

$$\frac{b_{t+1}^S}{p_t} = \psi_B (\phi_t - \phi) \quad (30)$$

where $\psi_B < 0$. This unconventional policy differs from the liquidity facilities studied in [Ashcraft, Gârleanu, and Pedersen \(2010\)](#), where the monetary authority following a tightening in margin requirements of securities lends against these

²⁶[Cui \(2016\)](#) analyzes the trade-off of issuing more liquid public debt financed via distortionary taxes.

securities at a lower haircut to alleviate the funding problems, similarly to the Term Asset-Backed Securities Loan Facility (TALF). Here the government does not directly relax the funding constraint but intervenes in the open market by providing a more liquid asset and changing the portfolio composition of entrepreneurs. In [Chen, Curdia, and Ferrero \(2012\)](#) the government also modifies the composition of outstanding liabilities, but increasing exogenously the supply of long-term bonds.

5 Numerical simulation

5.1 Parametrization

Table 2 reports the parameter values of the model calibrated at quarterly frequency. There are two sets of parameters. One is specific to the Irish economy for financial frictions (ϕ, θ) , the maturity of public debt (λ) and the coefficient for unconventional policy (ψ_B) . The other is standard in the literature of DSGE models. I calibrate the dynamics of ϕ_t using the liquidity shock identified via the narrative approach in Section 3.

The key parameter characterizing the financial frictions is the liquidity constraint ϕ . The steady state value of liquidity is 0.85, corresponding to one minus the haircut applied in the repo market by LCH Clearnet on 10-year Irish bonds before the crisis, which was 15%. The size of the liquidity shock is 0.40, equivalent to the rise in the haircut between the last quarter of 2010 and the first quarter of 2011, when the haircut jumped from 15% to 55% (see Figure 4). The persistence of the shock ρ^ϕ is 0.985, so that the ZLB binds for 20 quarters. This value also corresponds to the autoregressive coefficient of the liquidity shock ϕ_t estimated in

Table 2: Parametrization

Definition	Parameter	Value
<i>Preferences</i>		
Household discount factor	β	0.993
Relative risk aversion	σ	1.000
Inverse Frish elasticity	η	1.000
<i>Production and investment</i>		
Capital share of output	α	0.340
Investment adjustment cost	$\Gamma''(1)$	0.750
Probability of investment opportunity	γ	0.009
Depreciation rate	δ	0.027
<i>Nominal frictions</i>		
Price and wage calvo probability	$\zeta_\pi = \zeta_w$	0.750
Price and wage steady-state markup	$\delta_\pi = \delta_w$	0.100
<i>Financial frictions</i>		
Borrowing constraint	θ	0.300
Liquidity constraint	ϕ	0.850
Size of liquidity shock	$\Delta\phi$	-0.400
Shock persistence	ρ^ϕ	0.990
Steady-state of liquidity share	ls	0.430
Bond maturity parameter	λ	0.973
<i>Policy rules</i>		
Taylor rule inflation response	ψ_π	1.500
Taylor rule output response	ψ_y	0.125
Tax rule response	ψ_T	0.100
Baseline policy intervention	ψ_B	-0.2598

Note: The table shows the parameter values of the model.

an AR(1) process.

The other parameter characterizing the financial frictions θ describes the fraction of investment financed externally. Since entrepreneurs represent broadly the banking system in channeling resources to the production sector of the economy, I consider θ as the ratio of Irish banks' external finance over total assets, using aggregate banks' balance sheet data from the ECB Statistical Data Warehouse. I compute the average of this ratio for the period between 2000 and 2011, which is

0.3. The liquidity share in this economy is defined as:

$$ls_t = \frac{\phi q_t^L b_{t+1}^L}{\phi q_t^L b_{t+1}^L + q_t p_t k_{t+1}} \quad (31)$$

The nominator is the liquid part of public debt computed as the total of Irish government gross liabilities times the liquidity parameter. The denominator is equal to the value of the total productive capital. Data are taken from the OECD Economic Outlook. The average of this ratio during the period between 2000 and 2011 is 0.43, which is taken as the steady state value of the liquidity share.

The parameter λ pins down the duration of long-term bonds given by $(1 - \lambda\beta)^{-1}$. I set $\lambda = 0.973$ to match the average maturity of the Irish debt, which is 6.9 years (Eurostat (2013)). The calibration of the liquidity policy response parameter (ψ_B) is based on the actual ECB intervention on the Irish bond secondary market with the Expanded Asset Purchase Programme (APP), which is around 1% of Irish GDP.²⁷

There are two important differences with respect to Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017). In their calibration the size of the liquidity shock, based on the convenience yield of US Treasuries, is smaller (-0.218) and the coefficient of unconventional policy of the Federal Reserve in exchanging government liquidity for private financial assets is larger (-4.801), because its intervention was about 10% of GDP. As counterfactual I consider a scenario in which the unconventional policy is more aggressive and the size of intervention is 10% of GDP.

²⁷The share of national government bonds purchased by the ECB is based on its capital key rule. The monthly volume of Irish government bonds is around €0.7 bn. The calibrated value of the liquidity policy response parameter ψ_B does not consider other ECB's interventions such as the Security Market Programme and 3-year LTROs which swapped illiquid Irish bonds with public liquidity.

Other parameters are standard in the literature: the discount factor $\beta = 0.99$, the inverse Frish elasticity of labor supply $\eta = 1$, the relative risk aversion parameter $\sigma = 1$, the capital share $\alpha = 0.34$, the depreciation rate $\delta = 0.027$, $\Gamma''(1) = 0.75$ and the arrival rate of investment opportunity in each quarter $\gamma = 0.009$. For real and nominal rigidities, the degree of monopolistic competition in labor and product markets is calibrated symmetrically assuming a steady state markup of 10% ($\delta_p = \delta_w = 0.1$). The average duration of price and wage contracts is 4 quarters ($\zeta_p = \zeta_w = 0.75$). Concerning the monetary policy rule, the Taylor rule coefficient for inflation (ψ_π) and output gap (ψ_y) are 1.5 and 0.125, respectively. The government finances most of the intervention through emission of new short-term debt and transfers slowly adjust to the government net wealth position ($\psi_T = 0.1$).

5.2 Results

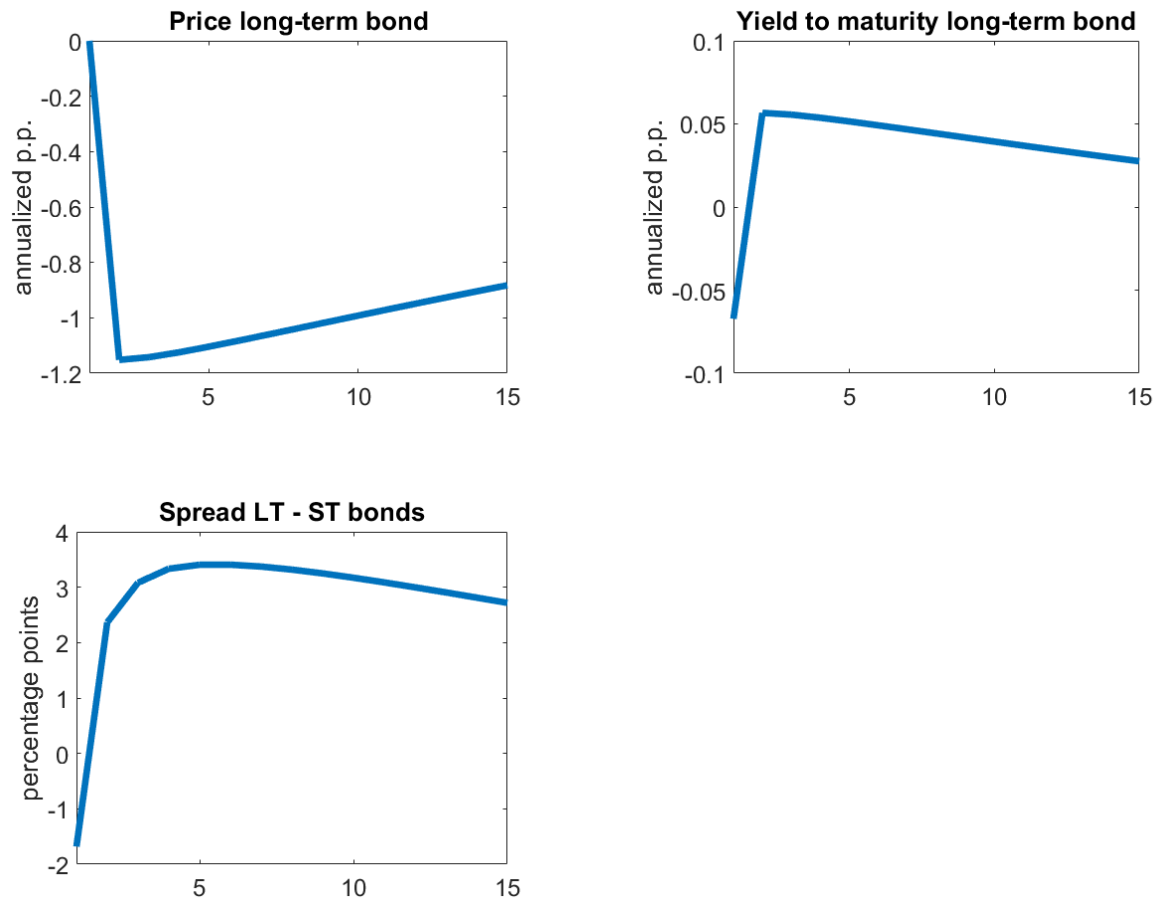
5.2.1 The impact of the liquidity shock

I first analyze the economy in which the government does not respond to the calibrated liquidity shock ϕ_t . As Figure 7 shows, the the price of long-term bond declines (1.1%) and yield to maturity (0.06%) increases in annualized percentage rates. Furthermore, the contraction in the liquidity of long-term bond leads to a “flight-to-liquidity” towards the more liquid short-term bond, as indicated by the rise in the spread between long-term and short-term bonds (3.3%).

Figure 8 shows the response of output, inflation, consumption and investment to the liquidity shock. Inflation is expressed in annualized percentage points, while the level of output, consumption and investment correspond to percentage deviations from the trend. The model suggests that the impact of the liquidity

shock on macroeconomic variables is large and persistent. Output and inflation fall by -12% and -11.5%.

Figure 7: Response of price and yield to maturity of long term bonds and the spread between long-term and short-term bond to the liquidity shock

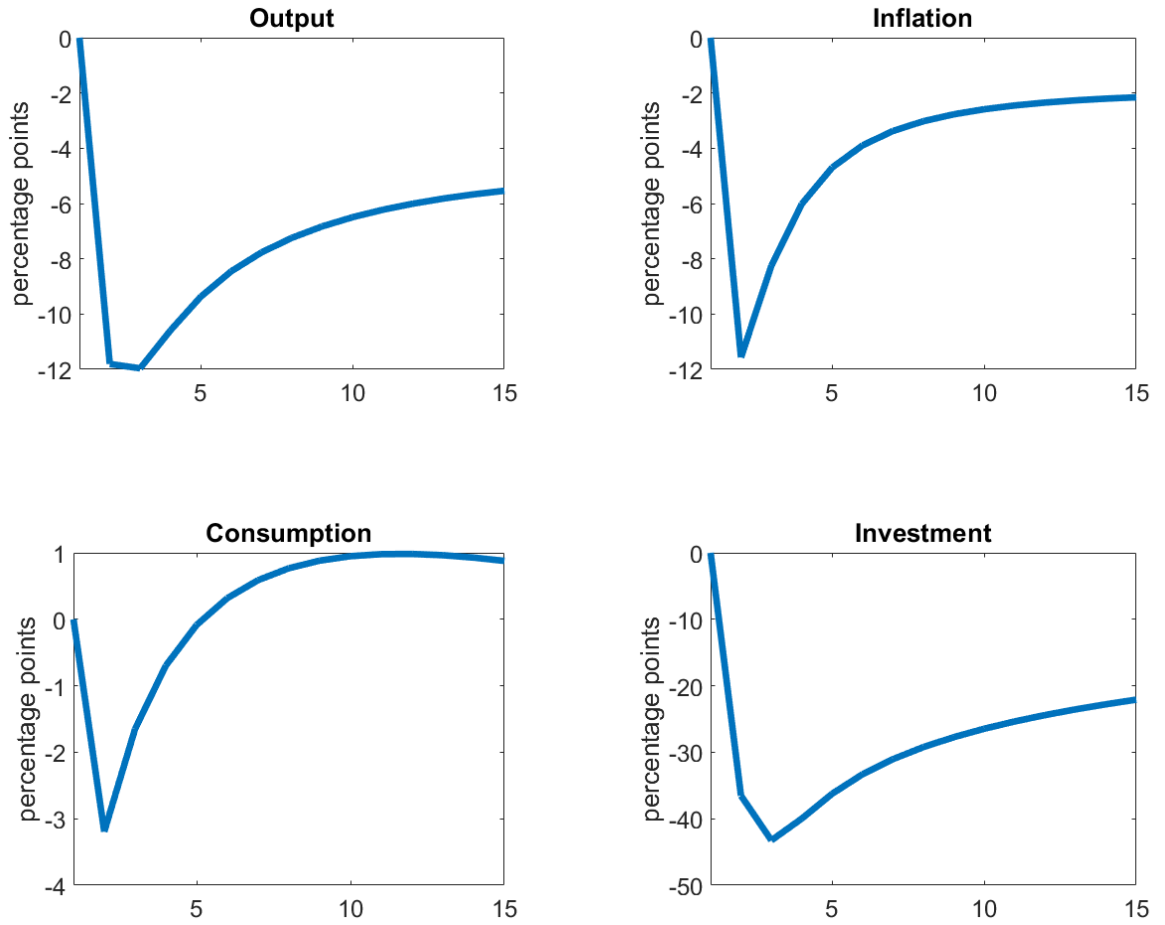


Note: The figure shows the response of price and yield to maturity of long-term bonds to the calibrated liquidity shock expressed in annualized percentage points and the response of the spread between long-term and short-term bond in percentage points.

Breaking down the drop in output to consumption and investment, the last row of Figure 8 shows that they reduce respectively by 3% and 40%. The contraction of liquidity impacts directly on investment by tightening the entrepreneurs' funding constraint and shrinking the liquidity they can obtain. The large drop in

investment is due to the magnitude of the liquidity shock. Calibrating the size of the liquidity shock with the same value as [Del Negro, Eggertsson, Ferrero, and Kiyotaki \(2017\)](#), investment falls to -14% and output and consumption decrease to -5% and -1.5%, respectively.

Figure 8: Response of output, inflation, consumption and investment to the liquidity shock



Note: The figure shows the response of output, inflation, consumption and investment to the calibrated liquidity shock. Output, consumption and inflation are expressed in the log-deviation from steady state in percentage points. Inflation is expressed in annualized percentage points.

The presence of nominal rigidities combined with the zero lower bound is a key element for the fall in consumption, because expected deflation leads to higher

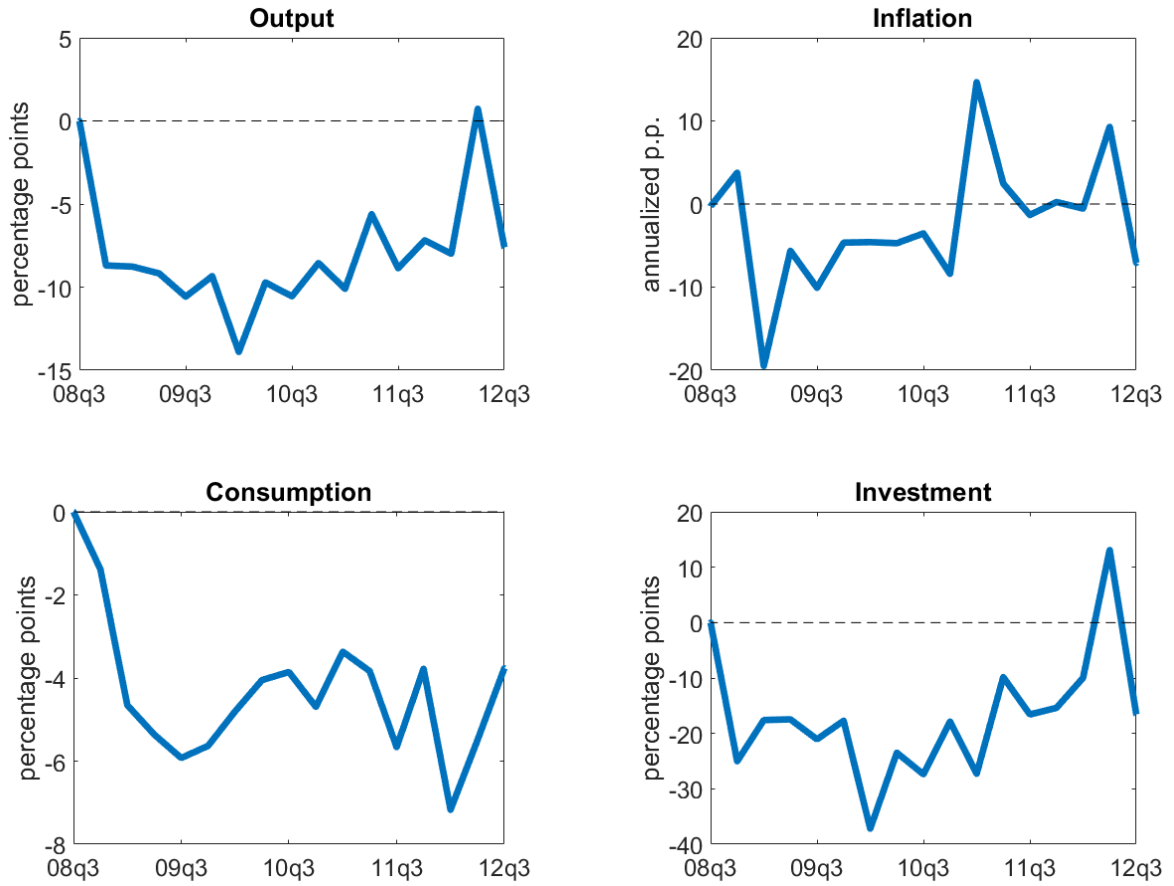
real interest rates; when the real interest rate declines the response of consumption becomes positive. With flexible prices the contraction in economic activity would lead to a negative real interest rate and a boost in consumption, as observed in [Kiyotaki and Moore \(2012\)](#) and [Del Negro, Eggertsson, Ferrero, and Kiyotaki \(2017\)](#).

The effect of a funding liquidity crisis generated with a drop in ϕ_t is analogous to a tightening of margin requirements of leveraged investors in [Gârleanu and Pedersen \(2011\)](#) and [Ashcraft, Gârleanu, and Pedersen \(2010\)](#), which increases the shadow cost of capital, raising the required returns of assets and lowering investment and output. In the model the liquidity shock has a smaller impact on the value and yield of long term bonds than in the empirical analysis. In [Kiyotaki and Moore \(2012\)](#) a negative liquidity shock on equity leads to an increase in equity price. [Shi \(2015\)](#) explains that this result is due to the rise in the demand for assets, including the liquid part of equity, which in their model is partly liquid. [Del Negro, Eggertsson, Ferrero, and Kiyotaki \(2017\)](#) shows that embedding nominal frictions and the ZLB allows equity price to drop following the tightening in the liquidity constraint on equity, but less than what observed in the data.

Figure 9 shows the changes in the data of output, inflation, consumption and investment for Ireland during the financial crisis. It reports the percentage deviation of output, consumption and investment from a linear trend estimated from 2000Q1 to 2012Q3, using the Hodrick Prescott filter, normalized to zero in 2008Q3. Consumption and investment are expressed in real terms and output is the sum of these two components. Inflation is the annualized percentage change in the GDP deflator in deviation from the 2% target of the ECB.

The model matches some properties of the data. Output contracts by -14%

Figure 9: Evolution of output, inflation, consumption and investment



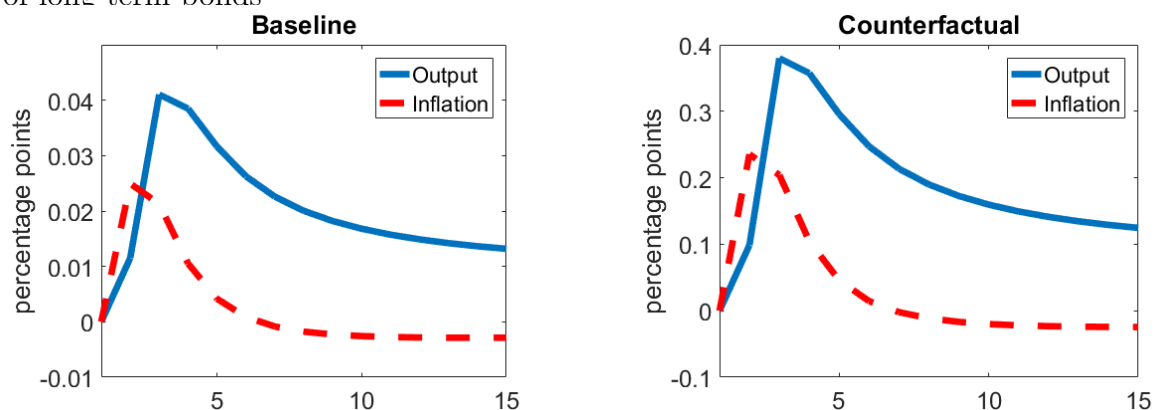
Note: The figure shows the evolution of output, inflation, consumption and investment in the data. Output, consumption and investment are in percentage deviations from a linear trend estimated with the Hodrick Prescott filter from 2000Q1 to 2012Q3 and are normalized to zero in 2008Q3. Consumption is the private final consumption expenditure divided by the deflator of private final consumption expenditure. Investment is the gross capital formation divided by the deflator of gross capital formation. Output is the sum of real consumption and real investment. Inflation is the annualized quarterly inflation rate of the GDP deflator. Data come from the OECD Economic Outlook N.101.

similarly to the model. Investment reduces much more than consumption, respectively -7% and -38%. Thus, the model overpredicts the fall in investment and underpredicts the fall in consumption, which is deeper and more persistent, but these differences are not far from what happened in Ireland during the crisis. Finally, in the data the drop in inflation is also very large (-20%).

5.2.2 The effects of the policy intervention

I consider now the case in which the government reacts to the drop in ϕ_t by issuing more short-term bonds, which are not subject to the liquidity constraint, under two scenarios: the baseline in which the policy intervention is calibrated for the APP in the Irish bond market and the counterfactual with a stronger intervention.

Figure 10: The effect of the unconventional policy on price and yield to maturity of long-term bonds

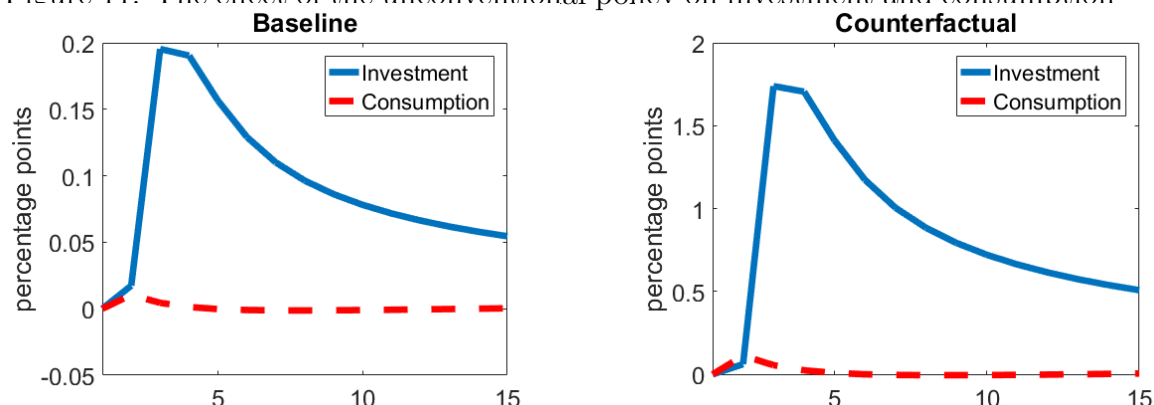


Note: This figure shows the difference between responses of investment (solid blue) and consumption (dashed red) to the calibrated liquidity shock with and without the policy intervention under the baseline scenario (left panel) and the counterfactual (right panel).

Figure 10 shows the gain in output and inflation for the two cases. In the first scenario the fall in output and inflation is reduced by only 0.04% and 0.03%, respectively (left panel). When we consider the counterfactual the unconventional policy leads to a more substantial gain for the economy: the decline in output and inflation is diminished by 0.38% and 0.25%.

Breaking down the gain in output by investment and consumption for the two scenarios, Figure 11 shows that almost all the gain of the unconventional policy comes from investment, which decline 0.2% less in the baseline and 1.8% less in the counterfactual as a result of the unconventional policy. By contrast, consumption

Figure 11: The effect of the unconventional policy on investment and consumption



Note: This figure shows the difference between responses of investment (solid blue) and consumption (dashed red) with and without the policy intervention under the baseline scenario (left panel) and the counterfactual (right panel).

is not affected by the policy in both cases.

The benefits of the government's intervention derive from the modifications of the investors' portfolio which become more liquid and allow investors to circumvent the liquidity constraint on long-term bonds with short-term bonds. However, the gains are delayed since the portfolio of investors is predetermined in the first period. The model suggests that this unconventional policy is successful in alleviating partly the contractionary effect of the liquidity shock but requires a strong policy intervention to produce quantitatively important results.

6 Conclusions

This paper has explored the liquidity channel of the Eurozone sovereign debt crises. It has shown that government securities play a key role as collateral in the secured interbank market, which is a primary source of funding for banks to meet their liquidity needs. Nevertheless, during the crisis repo haircuts on peripheral govern-

ment bonds substantially increased following the rise in their sovereign risk.

I identify a funding liquidity shock with the narrative approach by collecting information on variations in haircut set by LCH Clearnet, the largest European clearing house and I assess empirically the impact of a this shock on the Irish sovereign debt market with impulse response functions of BVAR and local projection models. Results suggest that the funding liquidity shock increases the government bond yield and CDS spread significantly, suggesting the presence of a negative liquidity spiral in the Irish government debt market.

I feed the funding liquidity shock into a DSGE model with liquidity frictions on long-term bond to simulate the impact of a rise in haircuts on macroeconomic and financial variables. The model predicts a fall in the value of illiquid long-term bond, a rise in its yield to maturity and a flight-to-liquidity from illiquid long-term bond to liquid short-term bond. Furthermore, the model exhibits a substantial fall in output, investments, consumptions and deflation. The contractionary effects of the liquidity shock can be alleviated by an unconventional policy consisting of issuing more short-term bonds to provide investors with a more liquid asset in order to relax their funding constraint, but requires a strong intervention.

Several other macroeconomic and financial shocks played an important role in the Eurozone crisis, including the losses in the banking system, the debt-deleveraging process of households and the austerity measures of the public sector, which contributed to the contraction of economic activity. However, the liquidity channel magnified the tension in the sovereign debt markets and reinforced the transmission to the real economy.

The theoretical model can be extended in several dimensions. One is to introduce the sovereign risk, which is absent in this model that focuses on a pure

liquidity channel. Following the literature on fiscal limit,²⁸ when the level of the debt approaches a stochastic threshold, the probability of default increases reducing the sovereign bond liquidity. This set up would allow to analyze the joint interaction between sovereign risk and liquidity. I leave this extension for further research.

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²⁸See [Bi \(2012\)](#), [Bi and Traum \(2012\)](#) and [Bi and Leeper \(2013\)](#).

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A Data Appendix

A.1 Definitions for repurchase agreements (repos)

A repo transaction is an agreement between two parties on the sale and subsequent repurchase of securities at an agreed price. It is equivalent to a secured loan, with the main difference that legal title of securities passes from the cash borrower to the cash lender, who may re-use them as collateral in other repo transactions. In order to protect the lender from the risk of a reduction in the value of collateral, repos involve overcollateralization and the difference between the value of the loan and the value of collateral is the haircut or initial margin. The haircut takes account of the unexpected loss that the lender may face due to the difficulty of selling the collateral security in response to a default by the borrower. Accordingly, it provides a measure of market liquidity of collateral from the standpoint of the lender and a measure of funding liquidity from the standpoint of the borrower since this determines the amount of cash that can be raised given the value of the collateral.

Figure 12 shows an example of a bilateral repo. At time t , the cash borrower (securities dealer, commercial bank, hedge fund) posts €100 securities as collateral and receives a €90 loan from the cash lender (commercial bank, investment fund, money market fund) with a haircut of 10%. At time $t+k$, the borrower returns the cash with an interest of 1.1% (the repo rate) and receives back the collateral. If repo is used to finance the purchase of a security, the haircut is equivalent to the inverse of the leverage. In order to hold €100 securities the investor can borrow up to €90 from the repo lender and must come up with €10 of its own capital, so the maximum leverage is 10. A rise in the haircut by 10 percentage points reduces

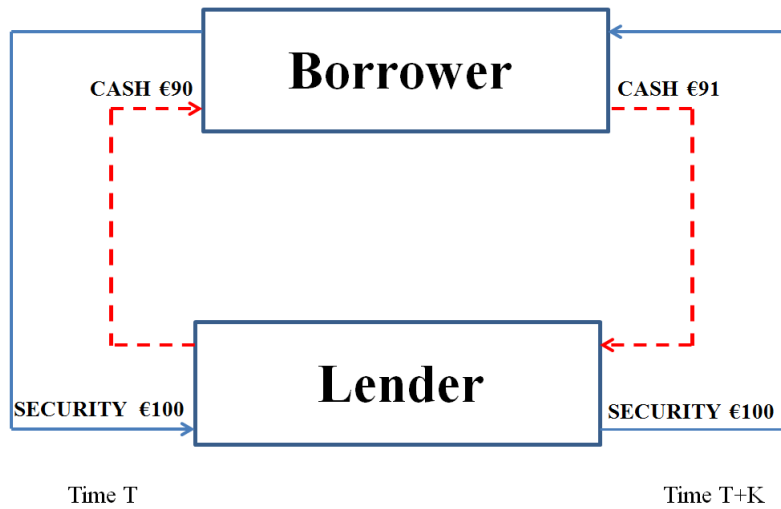


Figure 12: Bilateral repo contract

the borrower's funding to €80 and its leverage to 5.

According to the involvement of intermediaries between the lender and the borrower, repos can be distinguished as two types. In bilateral repos, the lender and the borrower transact directly with each other, selecting the collateral, initiating the transfer of cash and securities, and conducting collateral valuation. In tri-party repos, a third party intermediates the transaction, providing operational services to the parties, in particular the selection and valuation of collateral securities, but does not participate in the risk of transaction.

The determinants of haircuts vary according to the repo structure. In repos that are not cleared by a Central Clearing Counterparty (CCP), the haircut reflects mainly the creditworthiness of the borrower. Instead in repos involving a CCP which bears the counterparty credit risk, haircuts are settled on the basis of the CCP's internal rules and depend on the market risk of collateral.

A.2 Data on repos

Because of the lack of comprehensive information on the European repo market, I use different sources. First, Bankscope, which provide banks' balance sheet data at annual frequency showing the amount of repos and reverse repos held by credit intermediaries. It allows to compare different funding instruments, but lacks important breakdowns (such as counterparty, maturity and currency) preventing a more granular analysis and does not distinguish between repos in the interbank market from ECB monetary policy operations.

The table below shows the funding structure of the largest banks for which Bankscope report information on repos. I consider 2010 in order to avoid the 3-year LTROs implemented in 2011 and 2012.

Second, the European Repo Market Survey published semi-annually by the International Capital Market Association (ICMA) since 2001, which asks a sample of 67 banks in Europe for the value of their repo contracts that were still outstanding at close of a certain business date excluding repos transacted with central banks as part of official monetary policy operations. From the data of the European Repo Market Survey I subtracted reverse repos in order to focus the analysis on the liability side of banks' balance sheets. The survey reports information on the size and composition of the European repo market, including the type of repo traded, the rates, the collateral, the cash currency and the maturity. Third, the Euro Money Market Survey, an yearly survey published by the ECB since 2002 covering 101 banks, which breaks down the repo market to three segments: CCP-based, over-the-counter bilateral, and triparty.

Finally, I collected data on haircuts to 10-year government bonds applied by

Table 3: Funding structure of European commercial banks in percentage of total liabilities (2010)

Bank	Deposits	Interbank	LT debt	Repos
BNP Paribas	26.62	7.07	6.19	10.48
Barclays Bank Plc	23.41	5.89	9.89	13.26
Banco Santander	45.04	4.69	16.92	9.60
Société Generale	24.47	7.62	8.74	9.58
UBS AG	24.13	2.13	10.04	12.52
UniCredit SpA	42.99	10.83	16.96	3.39
Credit Agricole Corporate	14.43	10.63	0.70	5.61
Intesa Sanpaolo	30.84	10.32	27.99	1.99
Banco Bilbao	43.79	6.01	14.16	8.89
Commerzbank AG	34.32	12.01	13.72	7.13
Danske Bank	23.22	5.19	27.66	7.87
Skandinaviska Enskilda Banken	35.5	7.42	14.01	2.14
Bankia SA	36.78	3.43	29.6	11.56
Svenska Handelsbanken	29.21	8.06	30.27	0.49
Fortis Bank	41.18	7.68	5.50	4.30
Abbey National Treasury Services Plc	52.99	13.42	13.85	2.79
KBC	50.76	7.35	10.58	8.53
Banca Monte dei Paschi	32.35	9.41	24.56	9.86

Note: The table displays the structure of European commercial banks in percentage of total liabilities for 2010. Legend: Deposits = costumer deposits; Interbank = interbank deposits; LT debt = long-term debt; Repos = repurchase agreements.

Source: Bankscope. S

the LCH Clearnet, which is the largest European clearing house, and I use the Repo Clear Margin Rate Circulars to identify a funding liquidity shock with the narrative approach (see Appendix B.1).

B Econometric model

B.1 Identification of liquidity shock and statistical tests

Figure 13: Example of Repo Clear Margin Rate Circular

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Originating department:	Risk Management
Company Circular No:	LCH.Clearnet Ltd Circular No 2815
Service Circular No:	RepoClear: 160
Date:	24 March 2011
To:	All RepoClear Clearing Members

Management of Sovereign Credit Risk for RepoClear Service

In accordance with the Sovereign Credit Risk Framework and in response to the yield differential of 10 year Irish government debt against a AAA benchmark, LCH.Clearnet Ltd has revised the risk parameters for Irish government bonds cleared through the RepoClear service. The additional margin required for positions of Irish government bonds will consequently be increased to 35% for long positions; this amount will be adjusted for the current bond price. Short positions will pay a proportionately lower margin.

1. This decision is based solely on publicly available yield spread data and in no way represents a forward looking market view. LCH.Clearnet will continue to monitor yield spreads closely and keep the parameters under close review in accordance with the Sovereign Credit Risk Framework.
2. The additional margin will be reflected in a margin call/repayment on **Friday 25 March 2011**.
3. Report 74 (available on the LCH.Clearnet Member Reporting website) will detail any further changes in the margin levels charged under this framework.
4. This circular supersedes LCH.Clearnet Ltd Circular No 2746 dated 06 December 2010.
5. For further information please contact either Tom Chapman (tom.chapman@lchclearnet.com) +442074266335 or Lianne Arnold (lianne.arnold@lchclearnet.com) +442074267376

Christopher Jones
Executive Director, Head of Risk Management

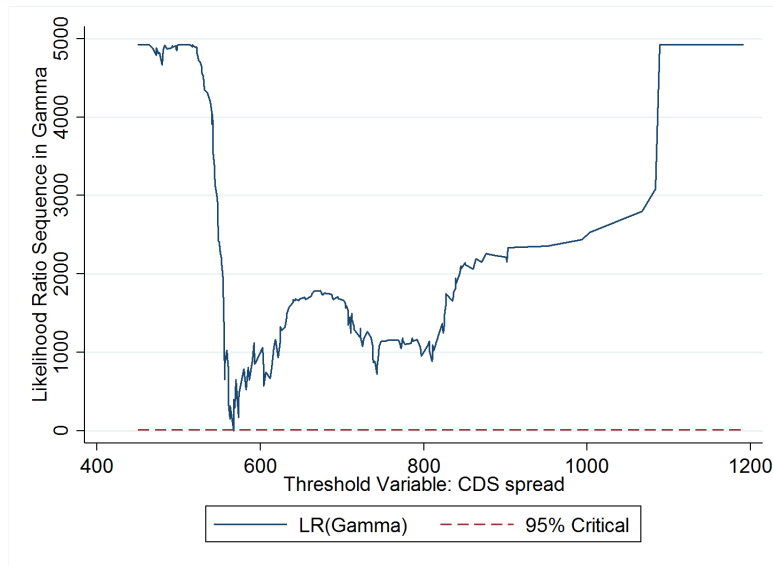
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Table 4: Granger causality tests

Hypothesis test	Result	F statistics	Critical value
<i>Do CDS spreads Granger-cause variations in haircut ?</i>	No	2.25	3.88
<i>Do yield spreads Granger-cause variations in haircut ?</i>	No	0.40	3.88

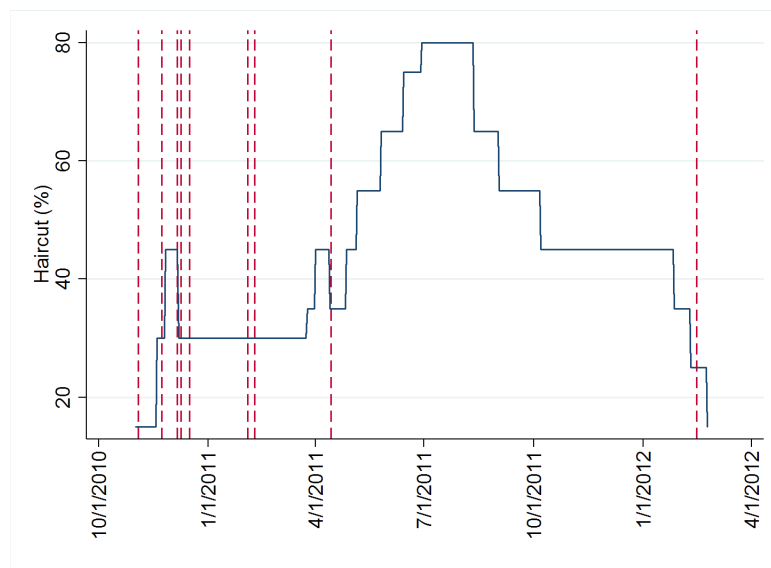
Note: The number of lags is selected using the Bayesian Information Criterion (BIC) considering a maximum of 10 lags. Tests are performed at the significance level of 0.05. If the F-statistics is lower than the critical value, we accept the null hypothesis that variable X does not Granger-causes variable Y.

Figure 14: Confidence interval construction for threshold



Note: The threshold test statistics is plotted for the regression $h_t = \alpha_0 + \alpha_1 CDS_t$. The figure displays a graph of the normalized likelihood sequences as a function of the threshold (CDS_t). The dotted line plots the 95% critical value.

Figure 15: Repo haircut and 500 bp threshold of CDS spread



Note: The blue line plots the haircuts on 10-year Irish government bonds applied the LCH Clearnet. The dash red vertical lines represent the episodes when the CDS spread of Irish government bonds breached the 500 basis point threshold.

B.2 Bayesian estimation

Given N different variables in a vector $y_t = (y_{1t}, \dots, y_{Nt})'$, consider the following VAR:

$$y_t = A_1 y_{t-1} + \dots + A_p y_{t-p} + u_t$$

where $t = 1, \dots, T$ and $u_t \sim (0, \Sigma_u)$. Each equation has $M = Np + 1$ regressors. By collecting the coefficient matrices in the $N \times M$ matrix $A = [A_c, A_1, \dots, A_p]$ and defining $x_t = (1, y'_{t-1}, \dots, y'_{t-p})'$ as a vector containing an intercept and p lags of y_t , the VAR can be written as

$$Y = X\Phi + A$$

where $Y = [y_1, \dots, y_T]'$, $X = [x_1, \dots, x_T]'$ and $A = [A_1, \dots, A_T]'$ are matrices of dimension $T \times N$, $T \times M$ and $T \times N$ respectively. Defining the following OLS estimations $\hat{A} = (X'X)^{-1}X'Y$, $\hat{a} = \text{vec}(\hat{A})$, $\hat{S} = (Y - X\hat{A})'(Y - X\hat{A})$, and $\hat{\Sigma}_u =$, we consider the non-informative version of the natural conjugate prior:

$$\alpha | \Sigma, y \sim N(\alpha^*, V^*)$$

$$\Sigma^{-1} | y \sim W(S^{-1}, T - K)$$

where $V^* = \Sigma_u \otimes (X'X)^{-1}$ and $a^* = \hat{a} + Q$. Q is the Choleski factor of V^* , i.e. $V^* = QQ'$.

C Additional Model Details and Optimality Conditions

C.1 Final and Intermediate Good Firms

Competitive final good producers combine differentiated intermediate goods Y_{it} , for $i \in [0, 1]$ into a single homogeneous final good Y_t , using a constant return to scale technology.

$$y_t = \left[\int_0^1 y_{it}^{\frac{1}{1+\delta_\pi}} di \right]^{1+\delta_\pi} \quad (\text{C.1})$$

where $\delta_\pi > 0$. They take input prices p_{it} and output prices p_t as given and choose y_{it} to maximize profits

$$p_t y_t - \int_0^1 p_{it} y_{it} di,$$

The solution to the profit maximization gives their demand for the generic i^{th} intermediate good

$$y_{it} = \left[\frac{p_{it}}{p_t} \right]^{-\frac{1+\delta_\pi}{\delta_\pi}} y_t \quad (\text{C.2})$$

The zero profit condition for competitive final-goods producers implies that the aggregate price level is

$$p_t = \left[\int_0^1 p_{it}^{-\frac{1}{\delta_\pi}} di \right]^{-\delta_\pi} \quad (\text{C.3})$$

Monopolistically competitive intermediate goods producers hire labor services

from households and rent capital from entrepreneurs to produce the intermediate goods choosing (i) the optimal amount of inputs, (ii) optimal price setting in case can adjust its price.

(i) The intermediate good producer i chooses the optimal amount of inputs to minimize the costs

$$r_t^K k_{it} + w_t h_{it}$$

subject to the production technology

$$y_{it} = a_t k_{it}^\alpha h_{it}^{1-\alpha} - \Omega \quad (\text{C.4})$$

where $\alpha \in (0, 1)$ is the share of capital, a_t is an aggregate productivity shock and Ω is fixed cost of production, k_{it} denotes the capital services and h_{it} the quantity of labor hired by the i^{th} intermediate-good producer. Defining $W_t = \frac{w_t}{p_t}$ the real wage, the first order condition implies that capital-labor ratio is independent of firm-specific variables

$$\frac{k_{it}}{h_{it}} = \frac{k_t}{h_t} = \frac{\alpha}{1-\alpha} \frac{W_t}{r_t^K} \quad (\text{C.5})$$

The Lagrange multiplier on the constraint is the real marginal cost which is also independent of firm-specific variables

$$mc_{it} = mc_t = \frac{1}{a_t} \left(\frac{r_t^K}{\alpha} \right)^\alpha \left(\frac{W_t}{1-\alpha} \right)^{1-\alpha} \quad (\text{C.6})$$

(ii) Intermediate good producers set prices p_{it} subject to [Calvo \(1983\)](#) scheme frictions. With probability $1 - \zeta_p$, the firm can reset its price and with probability

ζ_p cannot. By the law of large numbers, the probability of changing the price corresponds to the fraction of firms that reset the price, so each period a randomly selected fraction of firms $1 - \zeta_p$ choose the price \tilde{p}_{it} to maximize the present discount value of profit

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta \zeta_p)^{s-t} c_s^{-\sigma} \left[\frac{\tilde{p}_{it}}{p_s} - mc_s \right] y_s(i) = 0$$

subject to (C.2). The first order condition is

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta \zeta_p)^{s-t} c_s^{-\sigma} \left[\frac{\tilde{p}_{it}}{p_s} - (1 + \delta_\pi) mc_s \right] y_s(i) = 0$$

We focus on a symmetric equilibrium in which all firms choose the same price $\tilde{p}_{it} = \tilde{p}_t$. Let $\tilde{P}_t = \tilde{p}_t / p_t$ the optimal relative price. The first order condition for optimal price settings becomes

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta \zeta_p)^{s-t} c_s^{-\sigma} \left[\frac{\tilde{p}_{it}}{\pi_s} - (1 + \delta_\pi) mc_s \right] \left(\frac{\tilde{P}_t}{\pi_{t,s}} \right)^{-\frac{1+\delta_\pi}{\delta_\pi}} y_s = 0 \quad (\text{C.7})$$

By the law of large numbers, the probability of changing the price coincides with the fraction of firms who change the price in equilibrium. From the zero profit condition (C.3) inflation depends on the optimal reset price according to

$$1 = (1 - \zeta_p) \tilde{p}_t^{-\frac{1}{\delta_\pi}} + \zeta_p \left(\frac{1}{\pi_t} \right) \quad (\text{C.8})$$

Plugging (C.8) into (C.7) the price setting rule becomes

$$\left(\frac{1 - \zeta_\pi \pi_t^{\frac{1}{\delta_\pi}}}{1 - \zeta_\pi} \right)^{-\delta_\pi} = \frac{x_{1t}^p}{x_{2t}^p} \quad (\text{C.9})$$

where x_{1t}^p and x_{2t}^p are expected present value of real marginal cost and real marginal revenue as

$$x_{1t}^p = c_t^{-\sigma} y_t m c_t + \beta \zeta_p \mathbb{E}_t \left(\pi_{t+1}^{\frac{1+\zeta_\pi}{\zeta_\pi}} x_{1t}^p \right) \quad (\text{C.10})$$

$$x_{2t}^p = \frac{1}{1+\zeta_\pi} c_t^{-\sigma} y_t + \beta \zeta_p \mathbb{E}_t \left(\pi_{t+1}^{\frac{1}{\zeta_\pi}} x_{2t}^p \right) \quad (\text{C.11})$$

The evolution of the real wage is given by

$$\frac{w_t}{w_{t-1}} = \frac{\pi_{wt}}{\pi_t} \quad (\text{C.12})$$

where $\pi_{wt} = \frac{w_t}{w_{t+1}}$ is defined as the wage inflation. Since the ratio of capital-output is independent of firm-specific factors, the aggregate production function is

$$a_t k_t^\alpha h_t^{1-\alpha} = \int_0^1 y_{it} di = \sum_{s=0}^{\infty} \zeta_p (1-\zeta_p)^{t-s} \left(\frac{\tilde{P}_{t-s}}{\pi_{t-s,t}} \right)^{-\frac{1+\delta_\pi}{\delta_\pi}} y_t \quad (\text{C.13})$$

where $k_t = \int_0^1 k_{it} di$ and $h_t = \int_0^1 h_{it} di$. The effect of price dispersion is

$$\Delta_t = \sum_{s=0}^{\infty} \zeta_p (1-\zeta_p)^{t-s} \left(\frac{\tilde{P}_{t-s}}{\pi_{t-s,t}} \right)^{-\frac{1+\delta_\pi}{\delta_\pi}} \quad (\text{C.14})$$

Using (C.8), Δ_t can be expressed recursively

$$\Delta_t = \zeta_p \Delta_{t-1} \pi_t^{\frac{1+\delta_\pi}{\delta_\pi}} + (1-\zeta_p) \left(\frac{1-\zeta_\pi \pi_t^{\frac{1}{\delta_\pi}}}{1-\zeta_\pi} \right)^{1+\delta_\pi} \quad (\text{C.15})$$

The aggregate production function becomes

$$a_t k_t^\alpha h_t^{1-\alpha} - \Omega = \Delta_t y_t \quad (\text{C.16})$$

C.2 Labor market

The structure of the labor market is symmetric to that of the good market. Competitive labor agencies aggregate differentiated j labor inputs into a homogeneous single labor service h_t according to the technology

$$h_t = \left[\left(\frac{1}{1-\gamma} \right)^{\frac{\delta_w}{1+\delta_w}} \int_{\gamma}^1 h_t(j)^{\frac{1}{1+\delta_w}} dj \right]^{1+\delta_w} \quad (\text{C.17})$$

where $\delta_w > 0$. They choose h_{jt} to maximize their profits

$$w_t h_t - \int_{\gamma}^1 w_{jt} h_{jt} dj$$

subject to C.17 and taking wages specific w_{jt} as given. The first order condition yields the demand for the j^{th} labor inputs

$$h_t(j) = \frac{1}{1-\gamma} \left[\frac{w_t(j)}{w_t} \right]^{-\frac{1+\delta_w}{\delta_w}} h_t \quad (\text{C.18})$$

The aggregate wage index w_t comes from the zero profit condition for labor agencies

$$w_t = \left[\frac{1}{1-\gamma} \int_{\gamma}^1 w_{jt}^{-\frac{1}{\delta_w}} dj \right]^{-\delta_w} \quad (\text{C.19})$$

Labor unions represent all types of workers and set the wage rate w_{jt} for the specific labor input j subject to the Calvo scheme frictions on a staggered basis, taking as given the demand for their specific labor input. Each period, labor agencies

are able to reset the wage with probability $1 - \zeta_w$ and with probability ζ_w they cannot and the wage remains fixed. By the law of large number, the probability of changing the wage corresponds to the fraction of workers whose wages change. Households supply whatever labor is demanded at that wage. If labor agencies can modify the wage, they will choose the wage \tilde{w}_t to maximize

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta \zeta_p)^{s-t} \left[\frac{c_s^{1-\sigma}}{1-\sigma} - \frac{\xi}{1+\eta} \int_{\gamma}^1 h_s(j)^{1+\nu} dj \right] \quad (\text{C.20})$$

subject to (22) and (C.18). Focusing on a symmetric equilibrium in which all agencies choose the same wage, the first order condition for this problem is

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta \zeta_p)^{s-t} c_s^{-\sigma} \left\{ \frac{\tilde{W}_t}{\pi_{t,s}} - (1 + \delta_w) \frac{\omega \left[\left(\frac{\tilde{W}_t}{\pi_{t,s} W_s} \right)^{-\frac{1+\delta_w}{\delta_w}} H_s \right]^{\eta}}{c_s^{-\sigma}} \right\} \left(\frac{\tilde{W}_t}{\pi_{t,s} W_s} \right) h_s(j) = 0 \quad (\text{C.21})$$

from (C.19) the law of motion of real wage is

$$W_t^{\frac{1}{\delta_w}} = (1 - \zeta_w) \tilde{w}_t^{-\frac{1}{\delta_w}} + \zeta_w \left(\frac{W_{t-1}}{\pi_t} \right)^{-\frac{1}{\delta_w}} \quad (\text{C.22})$$

Using (C.22), (C.21) can be written

$$\left(\frac{1 - \zeta_{\pi} \pi_{wt}^{\frac{1}{\delta_{\pi}}}}{1 - \zeta_{\pi}} \right)^{-\delta_{\pi} + (1 + \pi_w) \eta} = \frac{x_{1t}^w}{x_{2t}^w} \quad (\text{C.23})$$

where x_{1t}^w and x_{2t}^w are the expected present value of marginal disutility of work and real marginal wage revenue as

$$x_{1t}^w = \frac{\omega}{(1 - \gamma) \eta} h_t^{1+\eta} + \beta \zeta_w \mathbb{E}_t \left(\pi_{wt+1}^{\frac{(1+\delta_w)(1+\eta)}{\delta_w}} x_{1t+1}^w \right) \quad (\text{C.24})$$

$$x_{2t}^w = \frac{1}{1 + \delta_w} c_t^{-\sigma} W_t h_t + \beta \zeta_w \mathbb{E}_t \left(\pi_{wt+1}^{\frac{1}{\delta_w}} x_{2t+1}^w \right) \quad (\text{C.25})$$

C.3 Capital-good producers

The creation of new capital is delegated to competitive capital-good producers who transform consumption goods into investment goods. They choose the amount of investment goods to maximize the profits taking the price of investment goods q_t^I as given

$$d_t^I = \left\{ q_t^I - \left[1 + \Gamma \left(\frac{i_t}{i} \right) \right] \right\} i_t \quad (\text{C.26})$$

The price of investment goods differ from the price of consumption goods because of the adjustment costs, which depends on the deviations of actual investment from its steady-state value. $\Gamma(.)i_t$ reflects the adjustment cost and $\Gamma(.)$ is a measure of technology illiquidity, capturing the difficulty to undo investment. We assume that $\Gamma(1) = \Gamma'(1) = 0$ and $\Gamma'' \left(\frac{i_t}{i} \right) > 0$. The first order condition for this problem is

$$q_t^I = 1 + \Gamma \left(\frac{i_t}{i} \right) + \Gamma' \left(\frac{i_t}{i} \right) \frac{i_t}{i} \quad (\text{C.27})$$

C.4 Market-clearing and equilibrium

The market-clearing conditions for composite labor and capital use are

$$h_t = \int_0^1 h_{it} di$$

and

$$k_t = \int_0^1 k_{it} di$$

The law of motion of capital is

$$k_{t+1} = (1 - \delta)k_t + i_t \quad (\text{C.28})$$

We consider the following identity equation between capital and net equity

$$k_{t+1} = s_{t+1} \quad (\text{C.29})$$

The resource constraint can be expressed as

$$y_t = c_t + \left[1 + \Gamma \left(\frac{i_t}{i} \right) \right] i_t \quad (\text{C.30})$$

Finally, considering the aggregate expression for d_t and d_t^I the investment function can be written as

$$i_t = \gamma \frac{r_t^K s_t + [1 + \lambda \phi_t q_t^L] \frac{b_t^L}{p_t} + q_t^S \frac{b_t^S}{p_t} + y_t - w_t h_t - r_t^K k_t + q_t^I i_t - i_t [1 + \Gamma \left(\frac{i_t}{i} \right)] - t_t}{q_t^I - \theta q_t} \quad (\text{C.31})$$

To solve the model, I define $B_{t+1}^L = \frac{b_{t+1}^L}{p_t}$ and $B_{t+1}^S = \frac{b_{t+1}^S}{p_t}$ as real long-term and short-term bonds. There are 5 endogenous state variables: the aggregate capital stock, the real long-term bond, the real short-term bond, and the real wage rate and the effect of price dispersion from the previous period $(k_t, B_t^L, B_t^S, W_{t-1}, \Delta_t)$. The recursive competitive equilibrium is defined as 9 endogenous quantities $(i_t, c_t, y_t, k_{t+1}, s_{t+1}, B_{t+1}^S, B_{t+1}^L, h_t, t_t)$ and 16 prices $(q_t, q_t^L, q_t^S, q_t^I, r_t^K, r_t, W_t, \pi_t, \pi_{wt}, mc_t, x_{1t}^p, x_{2t}^p,$

$x_{1t}^w, x_{2t}^w, \Delta_t, yd_t$) as a function of state variables $(k_t, B_t^S, B_t^L, W_{t-1}, \Delta_{t-1}, A_t, \phi_t)$, which satisfies the 25 equilibrium conditions (6, 7 21, 23, 24, 25, 27, 28, 29, 30, C.5, C.6, C.9, C.10, C.11 C.12, C.15, C.16, C.23, C.24, C.25, C.27, C.28, C.29, C.30,). Once all market clearing conditions and the government budget constraint are satisfied, the household budget constraint (22) is satisfied by Walras' Law.

C.5 Steady state

In the steady state Euler conditions are respectively

$$\beta^{-1} = \frac{r^K + (1 - \delta)q}{q} + \frac{\gamma(q - 1)r}{q(1 - \theta q)} \quad (\text{C.32})$$

$$\beta^{-1} = \frac{1 + \lambda q^L}{q^L} + \frac{\gamma(q - 1)}{q(1 - \theta q)} \frac{1 + \lambda \phi q^L}{q^L} \quad (\text{C.33})$$

$$\beta^{-1} = \frac{1}{q^S} + \frac{\gamma(q - 1)}{q(1 - \theta q)} \frac{1}{q^S} \quad (\text{C.34})$$

where in the steady state $p^I = 1$ because $\Gamma(1) = \Gamma'(1) = 0$. The non-arbitrage condition between short-term and long-term bonds in steady state implies

$$\frac{1}{q^S} = \frac{1 + \lambda q^L}{q^L} \quad (\text{C.35})$$

The capital-labor ratio is

$$\frac{k}{h} = \frac{\alpha}{1 - \alpha} \frac{w}{r^K} \quad (\text{C.36})$$

Since in the steady state all firms charge the same price, $\tilde{p} = 1$ and real marginal

cost is equal to the inverse of markup

$$mc = \frac{1}{a} \left(\frac{r^K}{\alpha} \right)^\alpha \left(\frac{w}{1-\alpha} \right)^{1-\alpha} = \frac{1}{1+\delta_\pi} \quad (\text{C.37})$$

Plugging these two equations into the production function at the steady state we obtain the capital-output ratio which is a function of the rental rate of capital.

$$\frac{y}{k} = \frac{r^K}{\alpha} \quad (\text{C.38})$$

Equation C.37 can be rewritten as a function of the rental rate

$$w = (1-\alpha) \left(\frac{a}{1+\delta_\pi} \right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{r^K} \right)^{\frac{\alpha}{1-\alpha}} \quad (\text{C.39})$$

In the steady state, the real wage is equal to a markup over the marginal rate of substitution between labor and consumption

$$w = (1+\delta_w) \frac{\omega h^\eta}{c^{-\sigma}} \quad (\text{C.40})$$

Assuming that $b^S = 0$ and considering $k = s$, the investment function in steady state is

$$i = \gamma \frac{r^K k + (1 + \lambda \phi q^L) b^L + \frac{\delta_\pi}{1+\delta_\pi} y - t}{1 - \theta q} \quad (\text{C.41})$$

The ratio of investment over capital is the depreciation rate

$$\frac{i}{k} = \delta \quad (\text{C.42})$$

The resource constraint is

$$y = c + i \tag{C.43}$$

Finally, from the government budget constraint the steady state tax is

$$t = [(1 + \lambda q^L) - q^L] b^L \tag{C.44}$$